Geostatistics - Variograms

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Summary

The spatial continuity of the variables studied must be quantified. In order to use the spatial correlation to improve inference at unsampled locations, several tools are defined to measure such spatial relationship. We will review the concepts of spatial continuity including correlograms, covariances, and variograms.

Pairs of points located at short distance show high correlation, while pairs farther away show a lower correlation. This intuitive idea is captured by the numerical tools described in this chapter.

We introduce the notion of anisotropy, that is, the fact that continuity depends on the direction, and discuss the conditions to infer a variogram, the possible interpretations of an experimental variogram, and the approach to modeling the variogram, and why this is needed.

1 Introduction

The notion of spatial continuity is intuitive. We assume that, because nature shows structure in most geological phenomena, things we find at one location in space should be similar at locations nearby. For example, if we find sand in a beach, we expect to find more sand a few steps away. Similarly, if we measure a concentration of lead at one location, originated from a pollution event, we expect to find more lead in nearby locations, since it is reasonable to expect those locations have been also affected by the pollution episode.

In mineral deposits, rock type units show structure, hence continuity in space. The concentration of an element within a rock type will tend to change slowly, since its deposition depends on the properties of the rock, such as its permeability. The hydrothermal fluids that permeated the rock and cooled to deposit the metals we seek, will follow permeable paths within the rock, generating a connected deposition.

We will measure this continuity by linking pairs of points that are separated by a distance vector $\mathbf{h}$. From there, we will define different measures of similarity or measures of dissimilarity and integrate this behavior for all possible distances and directions.

2 Comparing pairs of locations

The most intuitive notion of spatial correlation comes from comparing the variable at two locations separated by a distance $\mathbf{h}$. Notice that $\mathbf{h}$ is a vector with magnitude and direction that we can call lag separation distance. $\mathbf{h}$ can be “one meter in the East direction", for example. We can search and record all the pairs within a stationary domain that are separated by the vector **h**. Notice that the (second-order) stationarity assumption is needed, as we are implicitly saying that the difference in the grade in the two sample locations that form a pair is independent of where the pair is in the domain.

> Side note: the measures of spatial continuity that are presented next (variogram, covariance and correlogram) are two-point statistics and hence require a second order stationarity assumption for their inference.

If the variable is very continuous, the two elements of these pairs will be quite similar, even for large separation distances. On the other hand, a discontinuous variable will show significant differences between the two elements in the pair, even for short distances.

Collecting all the pairs found of samples separated by a vector **h**, we can plot them in a scatter plot (this is called h-scatter plot). The values found at the tail of the vector **h** can be labelled $z(\mathbf{u})$, while those found at the head of the vector, $z(\mathbf{u}+\mathbf{h})$ (see **Figure 1**). A statistical summary of the bivariate relationship is the correlation coefficient.

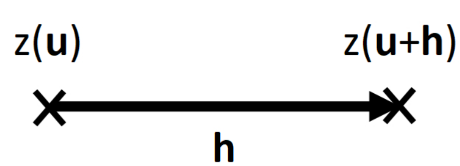

Figure 1: The lag separaton distance vector, showing its head and tail

The process can be repeated for different lag separation distances, and for each set of pairs of points, the correlation coefficient can be calculated. We should expect seeing a decreasing correlation, as the separation distance increases. In fact, at some point, we should expect to find pairs with no correlation.

The plot of the correlation coefficient versus the lag separation distance for the pairs, is called experimental correlogram. For distance **h**=0, the correlogram takes a value of 1, as the pairs are made of a sample with itself (z($\mathbf{u}_i$), z($\mathbf{u}_i$)). On the contrary, for large distances the correlogram tends to 0, as the pairs become uncorrelated when very far apart.

Let us rewrite the expression of the correlation coefficient using the tail and head values as variables:

$$\rho_{Z(\mathbf{u}),Z(\mathbf{u}+\mathbf{h})} = \frac{Cov_{Z(\mathbf{u}),Z(\mathbf{u}+\mathbf{h})}}{\sigma_{Z(\mathbf{u})} \cdot \sigma_{Z(\mathbf{u}+\mathbf{h})}} \quad (1)$$

where the covariance is:

$$Cov_{Z(\mathbf{u}),Z(\mathbf{u}+\mathbf{h})} = \frac{1}{n}\sum_{i=1}^{n}(z(\mathbf{u}_i)-m_{Z(\mathbf{u})})\cdot(z(\mathbf{u}_i+\mathbf{h})-m_{Z(\mathbf{u}+\mathbf{h})}) \quad (2)$$

Therefore, we can also measure the relationship with the spatial covariance, insead of the correlation coefficient.

Again, we can calculate and plot the covariance for different lag separation distances. The resulting plot is called experimental covariance.

From the equations presented above, we can see that both the correlogram and the covariance require knowing the means and standard deviations of the tail and head values of the pairs found. These values are not known, they are estimated from the available samples, thus the calculated values are just an approximation of the true correlogram and covariance (we make an abstraction here, where we pretend there is an underlying true random function that has a specific and unknown correlogram and covariance).

In order to avoid computing these moments (mans and standard deviations) prior to quantifying the spatial correlation, a different tool has been traditionally used in geostatistics: the variogram.

3 The variogram

The variogram (or the semi-variogram[1], to be precise) is the main tool used to quantify spatial correlation in geostatistics.

> The variogram is defined as half the average squared-difference between the value of the variable at locations separated by a distance **h**.

Unlike the correlogram or the covariance, the variogram measures how *different* the two locations are (on average), instead of how *similar* they are.

[1]Unless clearly stated, we will omit the word "semi-" when referring to the semi-variogram. Therefore, every time we mention the variogram, we are referring to *half* the average of the squared-difference of the values of the random variables at two locations separated by a specific distance and orientation.

Before presenting the practicalities of variogram calculation and modeling, we will go back to the idea of the random function, and work from the formal definition of the variogram to its estimator, and discuss the assumption for its inference:

- We call the underlying variogram, under the framework of random variables and the random function, the theoretical variogram (this is a mathematical abstraction).
- If we had exhaustive access to all the values of the regionalized variable in a domain, we would be able to compute the so called regional variogram (unlike the theoretical variogram, this is a quantity we could actually acquire, if we had infinite resources by exhaustively sampling the domain). The regional variogram can be seen as an output of a random drawing of the random function over the domain, which has the underlying theoretical variogram.
- In most cases, we only have a few measurements of the regionalized variable over the domain at sample locations. We can use these values to infer the sample variogram, also known as experimental variogram, which approximates the regional variogram.

3.1 The theoretical variogram

Recall that we defined a random function as a collection of random variables within a domain. We also discussed that, in order to make inference about the properties of this random function, we need the domain to be homogeneous. The best way to define this homogeneity is by stating that all random variables within the domain share some statistical and spatial properties. In particular, we expect that pairs of random variables show a similar relationship in all parts of the domain, that is, there are no trends in the values of the random variables, in their variability, or on their relationships. If this is the case, we call the random function stationary, and this allow us to pull together the data we have to make statistical inference about the properties of the random function.

The formal definition of the variogram is:

$$\gamma(\mathbf{h}) = \frac{1}{2} Var\left\{Z(\mathbf{u}) - Z(\mathbf{u}+\mathbf{h})\right\} \tag{3}$$

Notice that this assumes that the variance of the difference between two random variables, does not depend on their location within the domain, that is, does not depend on **u**, but only on their separation **h**.

Therefore, recalling that the variance of a random variable is defined as:

$$Var\{X\} = E\left\{[X - E\{X\}]^2\right\} \tag{4}$$

we see that the expression for the variogram can be expanded to:

$$\begin{aligned}
\gamma(\mathbf{h}) &= \frac{1}{2} Var\left\{Z(\mathbf{u}) - Z(\mathbf{u}+\mathbf{h})\right\} \\
&= \frac{1}{2} E\left\{\left[Z(\mathbf{u}) - Z(\mathbf{u}+\mathbf{h}) - E\{Z(\mathbf{u}) - Z(\mathbf{u}+\mathbf{h})\}\right]^2\right\}
\end{aligned}$$

If we assume the random function is stationary, then the mean of the random variables is constant everywhere in the domain, thus:

$$E\{Z(\mathbf{u})\} = E\{Z(\mathbf{u}+\mathbf{h})\} = m$$

Therefore, we can simplify the previous expression:

$$\begin{aligned}\gamma(\mathbf{h}) &= \frac{1}{2}E\left\{\left[Z(\mathbf{u})-Z(\mathbf{u}+\mathbf{h})-\left(E\{Z(\mathbf{u})\}-E\{Z(\mathbf{u}+\mathbf{h})\}\right)\right]^2\right\} \\ &= \frac{1}{2}E\left\{\left[Z(\mathbf{u})-Z(\mathbf{u}+\mathbf{h})-\left(m-m\right)\right]^2\right\} \\ &= \frac{1}{2}E\left\{\left[Z(\mathbf{u})-Z(\mathbf{u}+\mathbf{h})\right]^2\right\}\end{aligned}$$

In summary, under second-order stationarity, we can write the variogram as:

Theoretical variogram

$$\gamma(\mathbf{h}) = \frac{1}{2}E\left\{\left[Z(\mathbf{u})-Z(\mathbf{u}+\mathbf{h})\right]^2\right\}$$

3.2 The regional variogram

The regional variogram is basically the experimental variogram when all the possible samples in the domain are available. This is unrealistic, but it is interesting to think about it for a moment, before getting into the inference issues we will face when computing the experimental variogram.

Since we are assuming all locations are known within the domain for the calculation of the regional variogram, the result will use all the points that fall in the intersection of the domain and itself shifted by the separation distance vector **h** (see **Figure 2**).

In most cases we will consider unreliable the variogram computed for distances that are very large, since this leaves part of the domain unrepresented in the calculation of the regional variogram. That is, some points at the center of the domain will not participate in the calculation of the variogram, since they will neither be the head or the tail of a pair for the variogram calculation. This is represented in **Figure 3**. Both, the points in the regions A or C will be tails or heads of pairs used to compute the regional variogram. However, points in the region B will never be used.

> As a rule of thumb, the variogram is not computed beyond half the size of the domain.

3.3 The experimental variogram

In practice, we have a limited number of locations where samples are available. From this information, we need to infer the variogram, in the hope that it represents fairly the regional variogram. The regional variogram can be seen as an outcome of the underlying process controlled by the random function, which has a theoretical variogram. At the

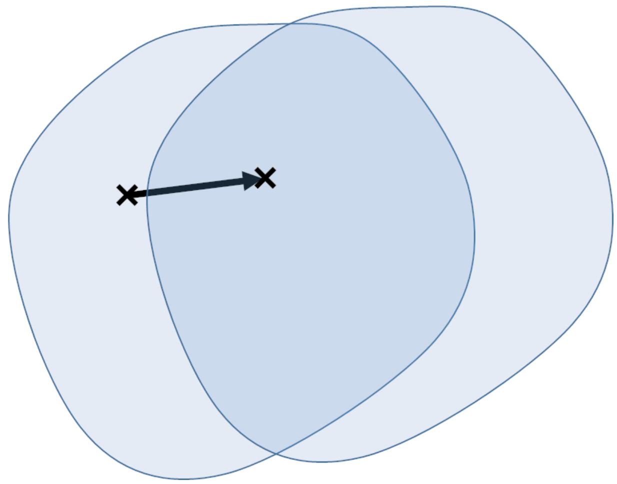

Figure 2: Domain intersection to compute the regional variogram for a lag **h**.

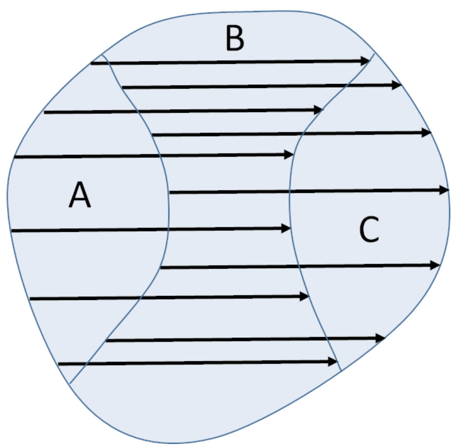

Figure 3: For large lag separation distances, the points at the center of the domain (region B) are neither head or tail of any pair, thus they are not represented in the regional variogram.

end, we will reconcile these views in a very practical way. The experimental variogram is used to infer the underlying theoretical variogram, which is just an abstraction to facilitate the use of statistics.

We can proceed by approaching the problem with a very simple statement: the best unbiased estimate of an expectation is the average of the available samples. This, of course assumes the sampling is representative of the entire domain and the variable is stationary.

Therefore, we can write:

Experimental variogram

$$\gamma(\mathbf{h}) = \frac{1}{2 \cdot N(\mathbf{h})} \sum_{i=1}^{N(\mathbf{h})} \left[Z(\mathbf{u}_i) - Z(\mathbf{u}_i + \mathbf{h}) \right]^2$$

So, the experimental variogram for a specific distance (and orientation) $\mathbf{h}$, is computed as half the average of the squared difference between the variable values for all the $N(\mathbf{h})$ pairs found.

Notice that we should have included a hat or asterisk in the $\gamma(\mathbf{h})$ to indicate it is an inferred (or estimated) value. We drop this to keep the notation simple.

The challenge is then to compute the variogram, interpret its behavior and, as we will see later, model it to know its value in any direction and for any distance.

4 Variogram calculation

To calculate the experimental variogram, we simply need to find all the pairs separated by the lag separation distance vector $\mathbf{h}$.

This seems as a trivial task, but from a computing point of view, it is an expensive operation, since it requires $\frac{n\cdot(n+1)}{2}$ comparisons. In each comparison, we need to determine if the distance is equal to $|\mathbf{h}|$ and if the direction of the separation of the pair is that of the vector $\mathbf{h}$.

A basic algorithm seudo-code is provided in **Algorithm 1**.

Experimental variogram algorithm

Data: Sample set: $\{z(\mathbf{u}_i), i = 1, \ldots, n\}$
Result: Variogram value for a lag separation distance: $\gamma(k \cdot \mathbf{h})$

```
Initialize N(k·h) and sum as 0
for i = 1,...,n do
    for j = i,...,n do
        Compute distance d_ij = ||u_i − u_j||
        if d_ij = k·h then
            if orientation of vector
             u_i − u_j = orientation of h then
                N(k·h) = N(k·h) + 1
                sum = sum + (z(u_i) − z(u_j))^2
            end
        end
    end
end
γ(k·h) = sum / (2·N(k·h))
```

Algorithm 1: Algorithm for variogram calculation

In the case of scattered data, we will rarely match the exact distance and orientation, therefore, the variogram is approximated by considering tolerances both in the distance and in the orientation of the vector separating the pair, with respect to the required lag separation vector.

Any vector direction can be defined by two angles:

- Azimuth: is the angle of the vector's projection in the XY plane, measured clockwise in the horizontal plane with respect to the North direction (this is a convention). The East has azimuth of 90°, the South direction of 180°, and the West has azimuth of 270°or -90°. A vertical vector does not need an azimuth to define its orientation, only a dip.
- Dip: is the angle over the vertical plane that includes the azimuth direction of the vector, measured with respect to the horizontal line. By convention, a positive sign is assigned if it points up, and a negative if it points down.

Therefore, to define a variogram with tolerances, we need to define tolerances over the direction and over the lag separation distance. Variograms are usually computed for a number of multiples of a basic lag. Tolerances in the lag separation distance usually refer to a proportion of this basic lag.

The typical parameters and tolerances used when computing an experimental variogram are:

- Parameters:
  - Basic lag: it is linked to the spacing of the data.
  - Number of lags (multiples of the basic lag): should be such that the basic lag multipled by the number of lags does not exceed half the size of the domain in that direction (recall the rule of thumb discussed under the regional variogram section).
  - Azimuth
  - Dip
- Tolerances:
  - Lag tolerance: typically 50% of the basic lag.
  - Azimuth angular tolerance: typically 15° or 22.5°. This value is added and subtracted to the required azimuth.
  - Dip angular tolerance: typically 15° or 22.5°. This value is added and subtracted to the required dip.
  - Half bandwidth: this is a maximum distance away from the lag separation vector, used to constrain the angular tolerances defined in azimuth and dip. Sometimes, a different bandwidth is used in the horizontal and vertical directions. It is usually defined according to the spacing of the data.

The main tolerances are illustrated in **Figure 4**.

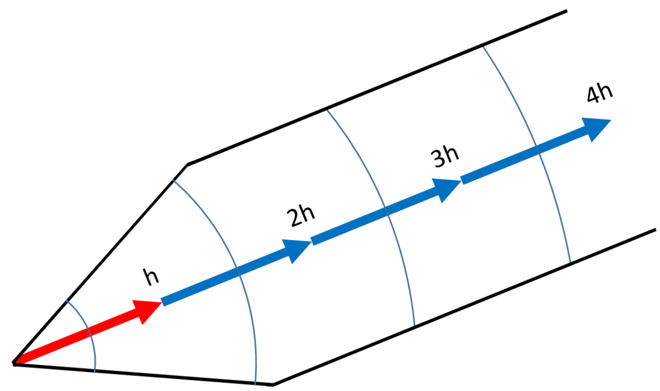

Figure 4: Illustration of variogram tolerances in a plane.

Experimental variograms are calculated in several directions, in order to explore the spatial continuity anisotropy. It is easy to imagine that the spatial continuity changes with direction. Considering gravity as a factor during any depositional event will likely generate a different behavior in the plane and in the vertical direction. In hydrothermal processes, the fluids containing the metals will permeate the rock through layers that are likely linked to the rocks formation, therefore some directions will be more permeable, thus prone to concentrating the metals, than others.

Anisotropies should reflect the understanding of the underlying geology. In most cases, the interpretation of the geological environment will dictate what are the likely principal directions of anisotropy, that is the directions where the maximum and minimum continuity are expected. Based on that expert knowledge, directional variograms will be calculated to confirm the anisotropy.

When geological interpretation is unclear, directional variograms can be used to investigate potential anisotropies, but care must be taken to avoid over interpretation of statistical fluctuations in the variogram. Also, one should be aware that the larger the tolerances, the least representative of a particular direction the variogram will be.

Variogram analysis (also called structural analysis) is often done by incrementally characterizing the variable. We may start by computing an omnidirectional variogram, which is an experimental variogram that only depends on the distance, but not on the orientation of the separation vector.

Similarly, the variogram can be computed over a main plane. This is done typically over the horizontal plane, but can also be done for a plane associated to a vein, for example, although this requires coordinates rotations. An onmi-horizontal variogram can be computed and compared to the vertical direction (which is a directional variogram, so there is no such thing as an omnivertical variogram).

Typical values and tolerances for different variograms are presented for reference, in **Table 1**. Notice that these values can be used as starting point, but the final definition of the parameters used depends on the sample data configuration, and the results of the structural analysis.

Type	Az	AzTol	Dip	DipTol	BandW
Directional	Any	15°or 22.5°	Any	15°or 22.5°	Depends
Horizontal	Any	15°or 22.5°	0°	15°or 22.5°	Depends
Omnidirectional	0°	180°	0°	180°	Depends
Omnihorizontal	0°	180°	0°	15°or 22.5°	Depends
Vertical	Any	180°	-90 °	15°or 22.5°	Depends

Table 1: Reference parameters for variogram calculation

The vertical variogram can be computed with a smaller lag separation distance, in case of vertical drilling. The spacing of the samples along the drillhole tends to be smaller than the spacing between drillholes in the plane. Thus, we can take advantage of this information to characterize the spatial continuity for short distances. In general we use multiples of the composite size, in the vertical direction.

Experimental variogram calculation is somehow iterative. We start by setting typical parameters for the lag (which depends on the spacing of the data), lag tolerance (50% of the basic lag), number of lags to compute (up to a distance that is half the size of the domain in that direction), azimuth, dip, angular tolerances and half bandwidth. From the results, we decide whether to change these parameters and tolerances, so that we balance “diluting” the directional nature of the variogram by increasing tolerances, with discovering the structure that exists in the random function. This means that, if with strict tolerance parameters we end up with a variogram that does not show structure, that is, we cannot see its nugget effect and how it increases with distance up to a plateau, then we need to iterate and increase the tolerances, understanding that this will hide some of the anisotropy.

5 Variogram interpretation

Once the variograms are available in multiple directions, comes the stage of interpreting the results.

Directional variograms should provide the detail of the behavior of the spatial continuity in different directions. If the phenomenon studied has clear anisotropies, these should be apparent from the shape of the experimental variograms in these directions.

For interpretation, we will focus our attention into the following features of the variograms:

- Short range: at short distances, we can identify the nugget effect and the behavior at short distances, that can be discontinuous (case of nugget effect), linear or parabolic.
- Long range: at long distances, we identify the sill, which is the variogram value at which the experimental variogram reaches a plateau and stabilizes, and the range, which is the distance where the variogram reaches the sill.
- Isotropy or anisotropy: If different directions show different variogram profiles, then we say the variogram is anisotropic in 2D or 3D. On the other hand, if the variogram shows a similar behavior, or in other words, if all directional variograms tend to overlap and follow the same shape, with similar nugget effect, range, sill and shape of the curve, then we say the phenomenon is isotropic.

Therefore, the interpretation stage requires plotting and comparing variograms in different directions, identitying their main features and trying to understand whether there are directions with a significantly different behavior than others (usually in terms of the range they reach or the sill they have).

Variograms from a stationary random function show a sill and this variogram value at long distances is actually equal to the variance of the variable.

In some cases variograms show particular features that can be easily interpreted:

- Trends can be detected because the variogram keeps increasing for larger separation distances, that is if the variable is not stationary, it will not stabilize at the expected sill value (the variance), but it will continue increasing beyond the variance.
- Cyclicity can be identified in the directional variogram and refers to cyclical depositional phenomena with a perfectly regular pattern. This is more commonly seen when analyzing time related data (time series).
- Geometric anisotropy is when in two different directions, the variogram has the same shape, but with different ranges.
- Zonal anisotropy is when in two different directions, the variogram has the same shape, but with different (apparent) sills.

These two types of anisotropy are very common and often come intermingled.

6 Variogram modeling

The sample variogram provides information about some directions and lag separation distances only. We will later discover that both for estimation and simulation, we need to know the variogram value for distances and orientations beyond those available from the experimental variograms. A model is therefore required.

Now, this model is going to fill the blanks of our sample variograms so that in any direction in the 3D space and for any distance, we can know the variogram value. We could simply interpolate between the available variogram values at specific distances and direction, however this is not allowed. It is not allowed because the variogram function will be used as a distance metric (in a mathematical sense) and thus requires to comply with some mathematical properties.

Without getting into the details of these requirements, we will just say that the practice is to use licit variogram models (see **Table 2**) that are known to comply with these conditions. We can "nest" these licit variogram models to fit the experimental variograms we have available and that is often enough for what we need.

Model	Equation	Notes
Nugget Effect	$\gamma(\mathbf{h}) = \begin{cases} 0 & \text{if } \mathbf{h} = 0 \\ C & \text{otherwise} \end{cases}$	The range is 0 and the sill is C
Spherical	$\gamma(\mathbf{h}) = \begin{cases} C\left(\frac{3}{2}\frac{\mathbf{h}}{a} - \frac{1}{2}\frac{\mathbf{h}^3}{a^3}\right) & \text{if } \mathbf{h} \leq a \\ C & \text{otherwise} \end{cases}$	The range is a and the sill is C
Exponential	$\gamma(\mathbf{h}) = C\left(1 - exp^{\frac{-3\mathbf{h}}{a}}\right)$	The practical range is a, where the variogram reaches 95% of the sill C
Gaussian	$\gamma(\mathbf{h}) = C\left(1 - exp^{\frac{-3\mathbf{h}^2}{a^2}}\right)$	The practical range is a, where the variogram reaches 95% of the sill C
Power	$\gamma(\mathbf{h}) = C\mathbf{h}^{\theta}$	Defined by a power $0 < \theta < 2$ and a positive slope C
Sine cardinal	$\gamma(\mathbf{h}) = C\left(1 - \frac{a}{\mathbf{h}} sin\frac{\mathbf{h}}{a}\right)$	The range is a and the sill is C

Table 2: Some licit variogram models. There are many more.

One last problem with variogram modeling is accounting for anisotropies. In a nutshell, what we do is approximate the anisotropic behavior with an ellipse or ellipsoid, by looking at how the ranges of the variograms change in different directions.

This requires simplifying any anisotropic behavior we may see in the data to characterize it with only the principal directions of anisotropy, that is, in 3D, three orthogonal directions that reflect the “best fit” of the directions with maximum and minimum continuity (range). The third direction is just the orthogonal to the two extremes.

We will try to better illustrate this approach with an example.

7 Example

We work over the unit defined by Rock Type 20.

This is a well defined unit, from a geological point of view, it concentrates the highest grades in the deposit and variations in average grade and local variance are fairly small at a scale reasonable for modeling (trend plots are not shown).

It is important to start by recalling the sample statistics of Cu grade within this unit (see **Table 3**).

In this deposit, there is information about the directions of anisotropy being approximately N30°E, N120°E, and vertical.

We start by analyzing in a plan view the spacing of the samples, and determine that they are approximately 30 to 35 m apart over the plane and composites are 12m in length. Since most drillholes are vertical, we can assume a vertical spacing of approximately 12m. These values are important to define the lags used for the variogram calculation and

Statistic	Value
Number of Data	1635
Mean	1.196
Std. Dev.	0.661
Coef. of Var.	0.552
Maximum	7.240
Upper Quartile	1.450
Median	1.080
Lower Quartile	0.783
Minimum	0.160

Table 3: Basic statistics for Cu in RT 20

the bandwidth tolerances.

We compute twelve horizontal directional variograms to confirm the anisotropy. We calculate these in intervals of 15°in the plane. The parameters used for these variograms are presented in **Table 4**.

The following colors are used:

- N0°E and N90°E → red
- N15°E and N105°E → black
- N30°E and N120°E → green
- N45°E and N135°E → blue
- N60°E and N150°E → yellow
- N75°E and N165°E → dark blue

The result of this calculation is presented in **Figure 5**.

We can see quite a lot of fluctuations in the curves (that should be increasing from a common nugget effect to the sill).

We update the tolerances to try to improve the results. The new tolerances are presented in **Table 5**.

The resulting variograms are shown in **Figure 6**.

Direction	Lag [m]	Lag Tolerance [m]	Azimuth [°]	Azimuth Tolerance [°]	Dip [°]	Dip Tolerance [°]	Bandwidth [m]
N0°E	30.0	15.0	0	22.5	0	15.0	20.0
N15°E	30.0	15.0	15	22.5	0	15.0	20.0
N30°E	30.0	15.0	30	22.5	0	15.0	20.0
N45°E	30.0	15.0	45	22.5	0	15.0	20.0
N60°E	30.0	15.0	60	22.5	0	15.0	20.0
N75°E	30.0	15.0	75	22.5	0	15.0	20.0
N90°E	30.0	15.0	90	22.5	0	15.0	20.0
N105°E	30.0	15.0	105	22.5	0	15.0	20.0
N120°E	30.0	15.0	120	22.5	0	15.0	20.0
N135°E	30.0	15.0	135	22.5	0	15.0	20.0
N150°E	30.0	15.0	150	22.5	0	15.0	20.0
N165°E	30.0	15.0	165	22.5	0	15.0	20.0

Table 4: Parameters for directional variogram calculation

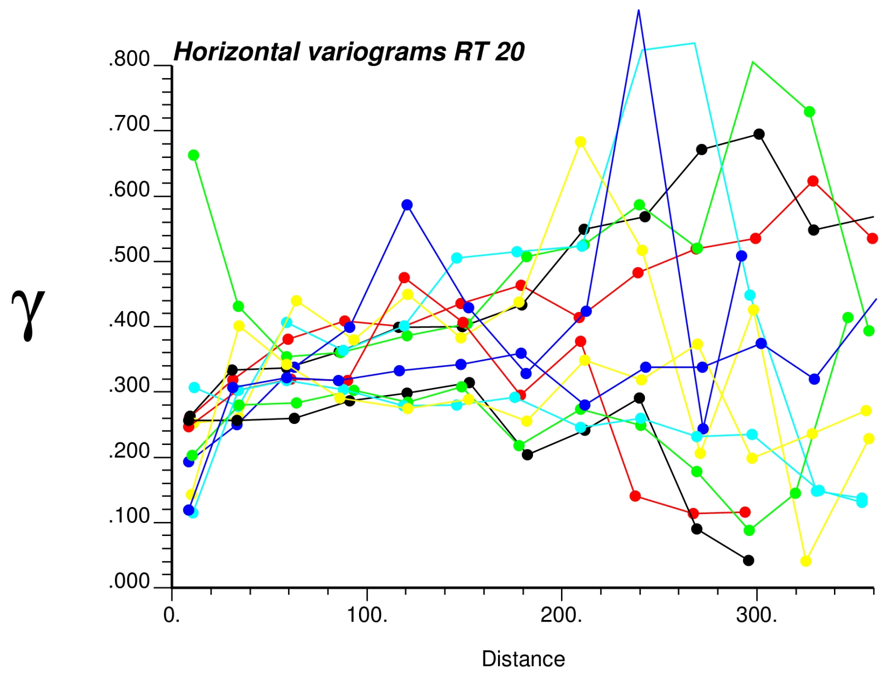

Figure 5: Horizontal directional variograms with the first set of tolerances.

Direction	Lag [m]	Lag Tolerance [m]	Azimuth [°]	Azimuth Tolerance [°]	Dip [°]	Dip Tolerance [°]	Bandwidth [m]
N0°E	35.0	17.5	0	22.5	0	15.0	30.0
N15°E	35.0	17.5	15	22.5	0	15.0	30.0
N30°E	35.0	17.5	30	22.5	0	15.0	30.0
N45°E	35.0	17.5	45	22.5	0	15.0	30.0
N60°E	35.0	17.5	60	22.5	0	15.0	30.0
N75°E	35.0	17.5	75	22.5	0	15.0	30.0
N90°E	35.0	17.5	90	22.5	0	15.0	30.0
N105°E	35.0	17.5	105	22.5	0	15.0	30.0
N120°E	35.0	17.5	120	22.5	0	15.0	30.0
N135°E	35.0	17.5	135	22.5	0	15.0	30.0
N150°E	35.0	17.5	150	22.5	0	15.0	30.0
N165°E	35.0	17.5	165	22.5	0	15.0	30.0

Table 5: Improved parameters for directional variogram calculation

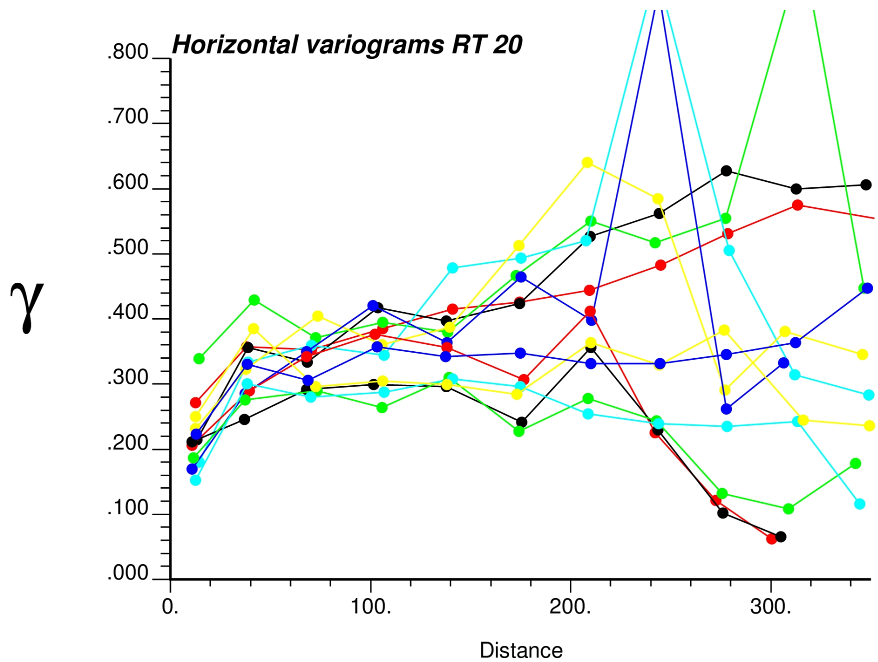

Figure 6: Horizontal directional variograms with improved tolerances.

We can see that, by increasing the lag and tolerances, the curves tend to be more stable. We identify that the curves in green for directions N0°E and N90°E show the extremes in terms of short range and long range variograms. It is clear that a zonal anisotropy occurs.

We now compute the vertical variogram and plot it together with the two main horizontal directions identified above. The parameters for computing the vertical variogram are shown in **Table 6**.

Direction	Lag [m]	Lag Tolerance [m]	Azimuth [°]	Azimuth Tolerance [°]	Dip [°]	Dip Tolerance [°]	Bandwidth [m]
Vertical	12.0	6.0	0	180.0	-90	15.0	20.0

Table 6: Parameters for vertical variogram calculation

The plot also displays the expected sill, which is the variance of the distribution (**Figure 7**).

The final step is to fit a model. As explained before, the model represents an ellipsoid where each nested structure is defined in 3D and parameterized by the ranges in the three principal directions. The axes of the ellipsoid must follow the main directions, so the variograms are rotated according to three angles:

- Angle 1: azimuth rotation
- Angle 2: dip rotation
- Angle 3: plunge rotation

After iterating to fit the model to the experimental variograms in the principal directions, the final model obtained is the one presented in **Table 7** and plotted in **Figure 8**.

Type	Sill	Angle 1	Angle 2	Angle 3	Range Y”	Range X”	Range Z”
Nugget	0.05						
Spherical	0.23	30	0	0	20	35	150
Spherical	0.0567	30	0	0	20	∞	150
Spherical	0.10	30	0	0	250	∞	150

Table 7: Parameters of the variogram model

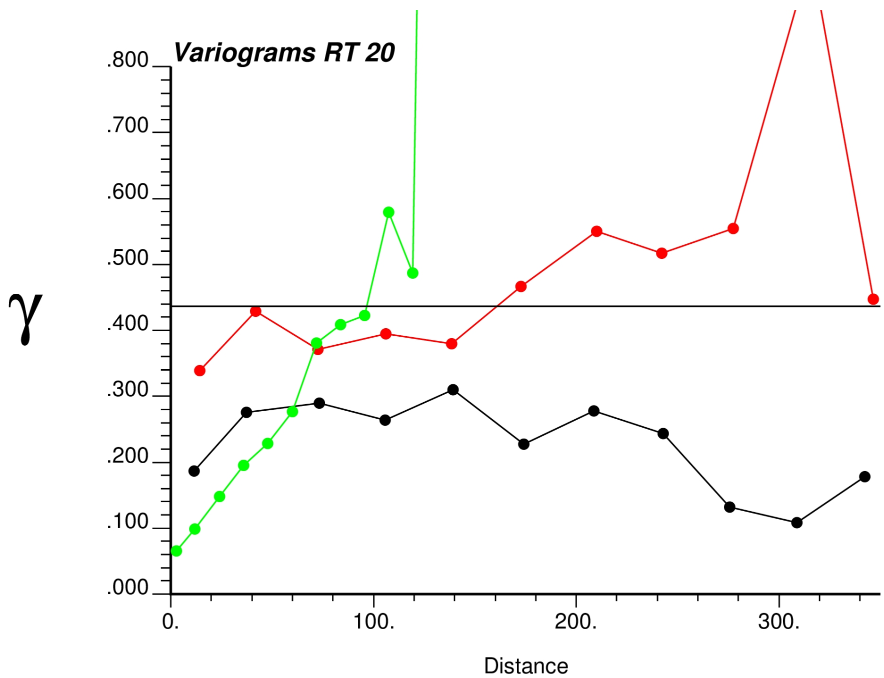

Figure 7: Directional variograms for the three principal directions.

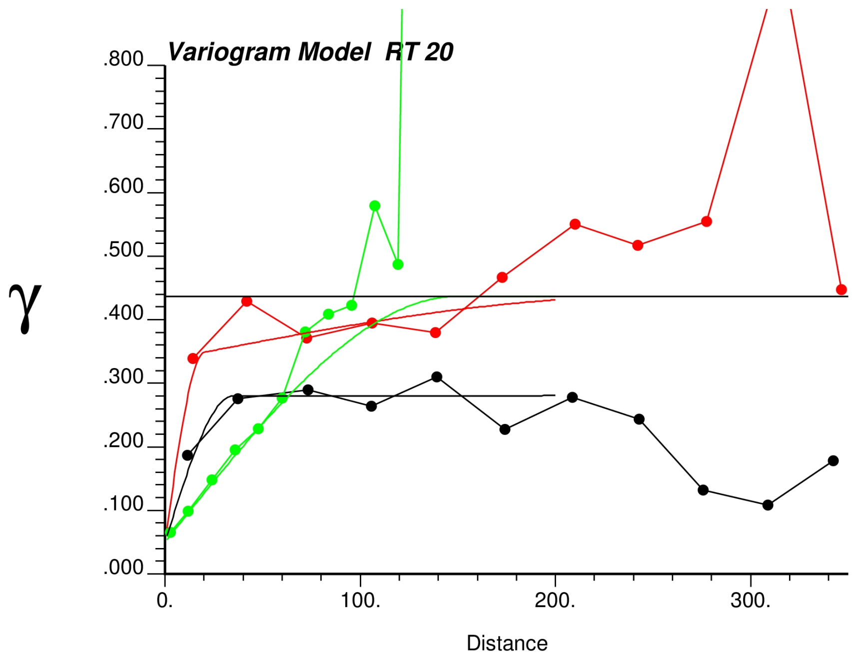

Figure 8: Fitted model in the principal directions of anisotropy.

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