# Geostatistics - Estimation - Practical Aspects

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## Summary

The practical implementation of estimation methods such as kriging requires defining the search parameters for the local definition of the neighborhood, and the conditions for the samples used for the estimation.

In this chapter, we discuss the practical aspects of setting up an estimation of a block model. We discuss how the search neighborhood and the stationarity assumption are linked, how the spacing of the samples is relevant to improve estimation, and how the result shows a smoothing effect.

We present some basic approaches to validate a block model, by showing the typical steps of visual inspection, statistical comparison and trend reproduction using swath plots.

# 1 Introduction

Implementing an estimation plan requires defining many parameters. Furthermore, several questions arise during the preparation of the estimation plan, such as:

- Are the parameters the best parameters that we can use to get a satisfactory block model?
- How can we compare two sets of parameters and choose the best plan?
- How do we validate or check a block model once completed?

In this chapter we will introduce the different approaches to answer these questions.

# 2 Estimation plans

An estimation plan is the collection of parameters used to define how the estimation will run. In particular, this includes the definition of the search neighborhood for samples, the selection criteria for the samples inside the search neighborhood, the estimation method, and the definition of which samples will be considered (this refers to using information from the domain of the estimation point or block, or combining information from different domains).

Typically, an estimation plan will require the following parameters:

- Estimation method
- Search radii for samples around the estimation location

- Minimum and maximum number of samples to use to estimate each location
- Constraints regarding number of quadrants or octants informed
- Constraints regarding maximum number of samples coming from the same drillhole or well
- Sample domains used to estimate specific block domains
- Constraints regarding the use of high valued samples

Notice that most estimation methods will not use all the data in the domain to estimate each location, but will constrain the data to a local neighborhood. This is justified by the fact that most variables have a variable mean over the domain, that is, first order stationarity (over the mean) is already hard to comply with, therefore, it is preferable to assume the variable is stationary over local neighborhood (quasi stationarity).

The size of the local neighborhood (defined by the search radii) will be tailored to match the anisotropy found in the continuity of the variable. However its extent will depend on how rapidly the mean values change in case a trend is present. Therefore, it is common to find search neighborhoods that are "corrected" to avoid mixing data from areas that are too different in mean to the estimation location.

The selection of the estimation method is really up to the user. Ordinary kriging has shown consistently robust performance under many different situations, provided there are enough samples to condition the estimation. Simple kriging is almost never used in practice, because of its stationarity assumption, however it forms the basis of other methods and is at the core of simulation techniques. Other types of kriging such as indicator kriging are sometimes used to deal with specific issues of the data, such as imposing different continuity for different ranges of values, or when continuity of high values is important for the transfer function considered over the data. A good example is when high permeability becomes a critical aspect of a project, such as with nuclear waste repositories.

Notice that if sufficient conditioning data are available and the variable has a range of continuity larger than the data spacing, then simple and ordinary kriging will give virtually the same result in interpolation areas. The man differences will be seen in areas where the data is extrapolated. This is where the mean (global in simple kriging, or local in ordinary kriging) has an impact.

The determination of the search radii will depend on the spatial continuity and spacing of the data. The goal is to find enough samples within the search neighborhood to generate a good estimation. The search radii tend to be defined according to the variogram ranges, however in some cases, we may want to have smaller radii (if we have enough samples) or larger (if we do not have enough samples and we need to make a reasonable implicit inference of the local mean in ordinary kriging).

The minimum and maximum number of samples for estimation also depends on the availability of samples. Typically, we will avoid estimating with less than 3 samples. ideally, we should have a larger number of samples surrounding the location, so we interpolate rather than extrapolate the value. A minimum of 8 samples is a reasonable number in 3D. For the maximum number of samples to be used, we must keep in mind that the number of samples define the size of the kriging system of equations. Therefore, if we have a block model with several million blocks to be estimated, it may take a large amount of time to perform at every location a kriging with a large number of samples. Recall that matrix inversion scales as $n^3$ (in other words, the computational complexity is $O(n^3)$, or in the most optimized algorithms, close to $O(n^{2.4})$), so increasing the number of samples by a factor of 2 increases computing time by a factor of 8 (or slightly over 5 if the most optimized matrix inversion is implemented).

The minimum number of quadrants or octants informed is used to ensure that extrapolation is avoided. Usually, information from at least 3 to 5 octants is expected to perform interpolation.

Additionally, the maximum number of samples per drillhole is also constrained to ensure not all the information comes from a single drillhole. Depending on the setting for the minimum and maximum number of samples to be used, this maximum per drillhole can be defined.

The definition of the domains from where the samples are drawn for estimating a block that belongs to a particular domain in the block model is often specified by a matrix, where the code associated to the sample, coming from the geological logging is linked to particular codes for the blocks in the model. **Table 1** shows an example where some samples are assigned to a single code in the block model. For example, samples with code 10 are assigned only to blocks with code 100. In other cases, samples are assigned to more than one code: samples coded 20 and 21 are assigned to both codes 200 and 210 in the block model. For block code 300, samples from units 30, 31 and 32 are combined.

Now, the type of boundaries between domains is also re-

| | | Block model code | | | | | |
|---|---|---|---|---|---|---|---|
| | | 100 | 110 | 200 | 210 | 300 | 400 |
| Sample code | 10 | Yes | No | No | No | No | No |
| | 11 | No | Yes | No | No | No | No |
| | 20 | No | No | Yes | Yes | No | No |
| | 21 | No | No | Yes | Yes | No | No |
| | 30 | No | No | No | No | Yes | No |
| | 31 | No | No | No | No | Yes | No |
| | 32 | No | No | No | No | Yes | No |
| | 40 | No | No | No | No | No | Yes |

Table 1: Example of correspondence between the sample code and the block code for estimation

quired (see **Table 2**). The type of boundary is indicated as hard, soft or transitional, and if transitional, the maximum search radius is indicated. This means that data from another domain can be used up to that distance from the boundary. For example, data from the units 200 and 210 can be used to estimate blocks from units 100 and 110, using a radius of up to 60*m* of the boundary between the respective domains. Notice that the reverse may not be treated in this manner: samples from domains 100 and 110 can be used to infer blocks from domains 200 and 210 but with a smaller radius, that is, they have less influence than when "crossing the boundary" in the opposite direction. This is often done to share samples between geologically similar units, but avoiding spreading high values into lower valued units. Soft boundaries basically indicate that no restriction is imposed over the samples of that domain and they can all be used to estimate a block in the current domain: this is what happens between units 100 and 110, and also between units 200 and 210. Hard boundaries indicate that no samples from the other domain can be used to infer a block in the current domain.

| | | To block code | | | | | |
|---|---|---|---|---|---|---|---|
| | | 100 | 110 | 200 | 210 | 300 | 400 |
| From block code | 100 | – | S | T(30m) | T(30m) | H | H |
| | 110 | S | – | T(30m) | T(30m) | H | H |
| | 200 | T(60m) | T(60m) | – | S | H | H |
| | 210 | T(60m) | T(60m) | S | – | H | H |
| | 300 | H | H | H | H | – | H |
| | 400 | H | H | H | H | H | – |

Table 2: Example of definition of boundaries for estimation

The final consideration for setting up the estimation plan is how to deal with high values (or outliers). When a high valued sample is identified within a domain, the options are:

- Remove from the data, if it is an error.
- Separate into a different population, if it is associated with other high valued samples in space.
- Maintain it into the population and "correct" its value, by capping to a maximum value selected. This is done through different methods, such as selecting a quantile associated to a break in the cumulative distribution (or the probability plot), or a gap in the histogram. Other methods in mining define the capping value depending on the metal quantity contained in deciles and percentiles, or evaluationg the "metal at risk".
- Control the influence during estimation using "high yield parameters", which essentially limit their spatial influence with a smaller radius and/or by capping.

# 3 Comparing estimation plans

Estimation plans can be compared before actually building the entire block model, by cross validation methods.

The basic idea of cross validation is to test the plan performance over locations with known values, that is, over sample locations.

## 3.1 Cross validation

The first approach to check the performance of an estimation plan is called cross validation. This is also known as leave-one-out validation.

A set of parameters for an estimation plan is defined, and then the plan is applied to estimate each one of the locations $\{\mathbf{u}_i, i = 1, \ldots, N\}$ where a sample is available. Obviously, the corresponding sample $Z(\mathbf{u}_i)$ is removed when its location $\mathbf{u}_i$ is estimated (otherwise kriging would return the exact sample value and the estimation would seem perfect!). The remaining $N-1$ samples are kept to apply the estimation plan, although only those within the search neighborhood that comply with the plan are used to estimate that location. This is repeated for all the sample locations, so the end result of this approach is a set of $N$ original samples $\{Z(\mathbf{u}_i), i = 1, \ldots, N\}$ and $N$ estimated values at the exact same locations $\{Z^*_{kriging}(\mathbf{u}_i), i = 1, \ldots, N\}$.

The approach can be repeated with different estimation plans to assess the performance of each set of parameters and decide which one gives the desired results.

## 3.2 Jack-knife

The second approach to analyze an estimation plan is jack-knife. It entails splitting the data into two subsets, and using one subset to estimate the locations of the other. Then a comparison of the results similar to that of cross-validation can be done.

Jack-knife is sometimes done when different drilling campaigns exist. In this case, care should be taken to ensure that both campaigns are comparable, that is, samples are unbiased and have the same precision. This has to do with sampling procedures and sample quality, which is a requirement for any good model.

## 3.3 Statistical analysis of validation results

If the estimation plan performs well, the estimated values will resemble the actual sample values. Now, this is formally measured through a statistical analysis of the errors. For each sample location, we compute the error and standardized error:

$$Err(\mathbf{u}_i) = Z^*_{kriging}(\mathbf{u}_i) - Z(\mathbf{u}_i) \tag{1}$$

$$StdErr(\mathbf{u}_i) = \frac{Err(\mathbf{u}_i)}{\sigma_{kriging}(\mathbf{u}_i)} \tag{2}$$

where $Z^*_{kriging}(\mathbf{u}_i)$ has been obtained by cross validation or jack-knife, and $\sigma_{kriging}(\mathbf{u}_i)$ is the corresponding kriging standard deviation.

The following statistics are evaluated:

- Mean of the errors (global bias): if the estimation plan is unbiased, the mean of the errors should be equal to 0. This reflects global unbiasedness. Furthermore, the errors can me mapped in space and contoured to identify issues of stationarity and proportional effect.
- Variance of the errors (precision): if the estimation plan performs well, the errors should have a small variance, as compared to the actual magnitude of the samples. The smaller the variance of the error, the more precise is the estimation.
- Histogram of the errors (normality of the errors): the histogram of the errors can be built to check if the errors are normally distributed, and to identify unusually high or low errors.
- Variance of standardized errors (variogram fit quality): the standardized errors are the errors divided by the kriging standard deviation. Their variance should be equal to one, if the variogram fit is appropriate. Unusually high or low standardized errors should be checked to understand the reason or such a significant departure. These can be identified as those with basolute value larger than 3 (which in a normal distribution have a probability of less than 0.3%) or 4 (which have a probability of less than 0.006%).
- Relationship between the estimated and the true value (conditional bias): a scatter plot can be created to analyze the relationship between the estimated and true values. This allows inspecting the smoothing effect of the estimation, potential outliers, and the correlation coefficient can be determined.

## 3.4 Selecting an estimation plan

Cross validation or jack-knife can help understanding the performance of different estimation plans, but we now need to select one to build the final model.

The different criteria refer to different aspects of the estimator: bias, precision, and conditional bias. The estimation plans may perform differently in these aspects, and in many cases, one plan may not outperform the other in all the aspects. The modeler must prioritize which performance metrics are more important.

Cross validation has some drawbacks. It is overly optimistic, as most of the time, very close samples will be available to the location estimated. In particular, the neighboring samples within the same drillhole will be at a distance equal to the composite length. Therefore, most points estimated will be supported by two very close samples. Therefore, the estimation will not vary much by changing the kriging plan. One possibility to make it more realistic, is to constrain the samples to a minimum distance, so samples that are immediate neighbors of the sample locations estimated are excluded. Another option is to exclude the full drillhole from the estimation (leave-drillhole-out validation), in which case, we will be overly pessimistic, as the closest samples will be unrealisticly far from each estimation location.

Jack-knife, on the other hand, also has some drawbacks. Since the dataset is split into two subsets, it provides insight in a case that is different than the one at hand, since the sample density is different. Therefore, it gives a pessimistic idea of how kriging will perform. Issues of data quality are also important when two different datasets are used iin jack-knife.

# 4 Validation of block model

The final estimated block model must be validated against the samples. Any other knowledge about the variable can be used to analyze the final model, such as expected trends or zonations, continuity or connectivity of values, etc.

Typical validations include:

- Visual inspection of the model: plan views and cross section with the block values and the samples displayed over the same slices and with the same color scale, can help detecting gross errors in the model. One should expect to see values close to the sample values in the corresponding blocks where the samples are located. Any unusual artifact can be identified by visual inspection. This validation is very important and should not be skipped: always look at the model!
- Statistical comparison: summary statistics of the samples and block values should be presented and compared. The means should be similar, since kriging is an unbiased method. The variance of blocks should be smaller than the variance of the samples, due to the smoothing effect. Notice that for the purpose of comparison, two issues will arise. Firstly, samples should be representative of the domain, therefore declustering weights may be required to obtain the correct statistics. Secondly, it is normal for the estimation method to extrapolate far beyond the samples, and depending on the method and local trends, this may bias the block statistics. Therefore, it is recommended to make a comparison constraining the volume over which blocks and samples are used, to ensure they are comparable.

- Swath plots: these are plots of conditional means of both the sample values and the block estimated values for slices along the main coordinates (although rotated coordinates can also be used). They help to verify if the block model follows the trends of the samples. We should expect to have more variability in the sample conditional means, but the general trend should be followed by the block estimated values. It is always a good idea to report for each slice, the number of samples available and the number of blocks estimated. Larger differences will likely be more frequent in slices with scarce samples.

Unsatisfactory results should not be disregarded. If there is any concern about the model, it should be revised and validated again. It is usual practice, to compare the kriging estimates with other simpler interpolation methods such as inverse distance and nearest neighbor estimation. The latter provides declustered statistics and the exact same set of blocks can be included in the comparison.

# 5 Example

We now present an example of kriging, using the data shown previously. We focus on unit 20 where most of the data are and start by performing a cross validation.

## 5.1 Cross validation analysis

Two plans are defined to illustrate the analyses done with the results of cross validation.

The cross validation results include for each sample:

- Coordinates X, Y and Z.
- True sample value, $Z(\mathbf{u}_i)$.
- Estimated value at sample location, $Z_K^*(\mathbf{u}_i)$. Notice that this estimated value is computed excluding the sample at location $\mathbf{u}_i$.
- Estimation variance at the sample location, $\sigma_K^2(\mathbf{u}_i)$.
- Error, which corresponds to the difference between the estimated value and the true value:

$$Err = Z_K^*(\mathbf{u}_i) - Z(\mathbf{u}_i) \tag{3}$$

From these statistics, we can compute, for each sample location, the standardized error as the error divided by the square root of the estimation variance:

$$StdErr = \frac{Err}{\sigma_K(\mathbf{u}_i)} \tag{4}$$

### Leave-one-out cross validation

We start by comparing two plans leaving only the sample out during cross-validation (see **Table 3**). We will later test what happens if we remove the entire drillhole.

The results for Plan 1 are summarized in **Figures 1** and **2**, while Plan 2 is summarized in **Figures 3** and **4**.

| Parameter | Plan 1 | Plan 2 |
|---|---|---|
| **Method** | OK | OK |
| **Min samples** | 2 | 24 |
| **Max samples** | 4 | 48 |
| **Max per octant** | not used | not used |
| **Search radii** | 50m | 150m |

Table 3: Parameters for leave-one-out cross validation

Results show the expected behavior of the estimate. The comparison is summarized in **Tables 4** and **5**, for Plans 1 and 2, respectively. The estimates are unbiased (mean errors of 0.0037 and 0.0050) and in both cases the variance is reduced due to the smoothing effect of kriging: the standard deviation of the true grades is 0.6608, while that of the estimated from Plan 1 is 0.5554, and for Plan 2 0.5212. The minimum and maximum estimates are closer to the mean than in the distribution of true grades. The standard errors are centered close to 0 and the standard deviations of the standard errors are close to 1, which indicates a reasonable fit of the variogram model. Some standard errors are very large in absolute value and could be checked to ensure they are due to an anomalous high grade value (which is hard to estimate even from close samples, as these are always lower values).

The correlation coefficient between the true and estimate is quite high and there is little difference between the two kriging plans. It is almost 0.8, which means about 64% of the variability can be explained by the data and the krigng model does not capture the remaining variability, and therefore, cannot predict the value with higher precision. Notice that the total sill of the varogram was 0.4367 and the nugget effect was 0.05, representing 11% of the total variance. Therefore, the variability can be described as:

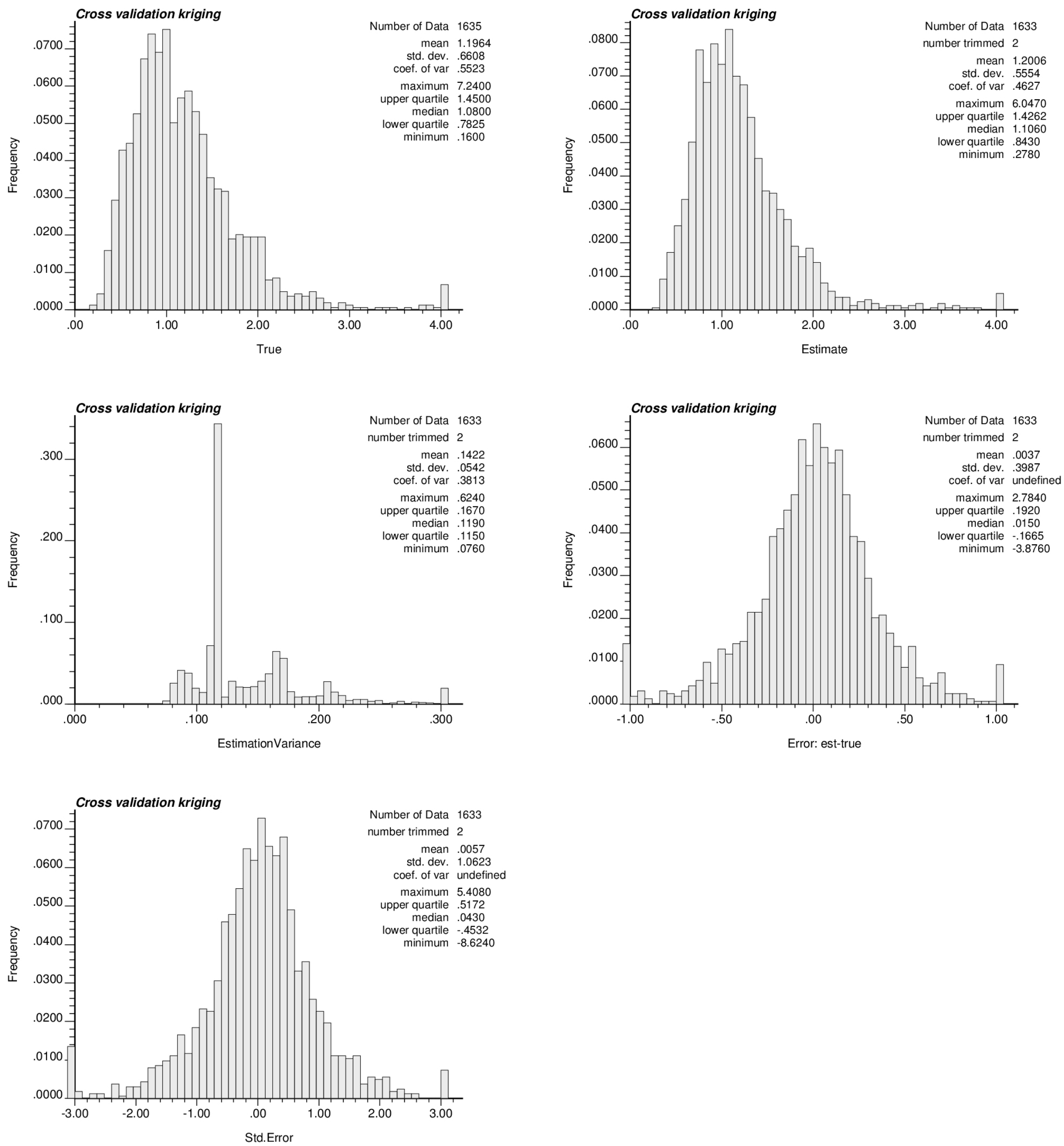


Figure 1: Leave-one-out cross validation results (histograms) for Plan 1 described in **Table 3**.

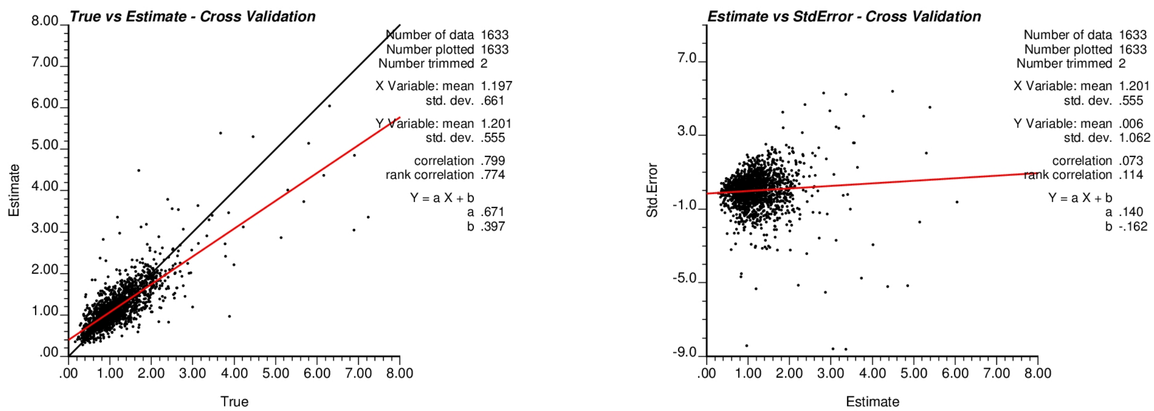


Figure 2: Leave-one-out cross validation results (scatter plots) for Plan 1 described in **Table 3**.

| | **True** | **Estimate** | **EstVar** | **Error** | **StdErr** |
|---|---|---|---|---|---|
| **Number of Data** | 1635 | 1633 | 1633 | 1633 | 1633 |
| **Mean** | 1.1964 | 1.2006 | 0.1422 | 0.0037 | 0.0057 |
| **Std. Dev.** | 0.6608 | 0.5554 | 0.0542 | 0.3987 | 1.0623 |
| **Coef. of Var.** | 0.5523 | 0.4627 | 0.3813 | undef | undef |
| **Maximum** | 7.24 | 6.05 | 0.62 | 2.78 | 5.41 |
| **Upper Quartile** | 1.45 | 1.43 | 0.17 | 0.19 | 0.52 |
| **Median** | 1.08 | 1.11 | 0.12 | 0.02 | 0.04 |
| **Lower Quartile** | 0.78 | 0.84 | 0.12 | -0.17 | -0.45 |
| **Minimum** | 0.16 | 0.28 | 0.08 | -3.88 | -8.62 |
| **Correl. Coef. True vs Estimate** | | | | 0.799 | |
| **Correl. Coef. Estimate vs StdErr** | | | | 0.073 | |

Table 4: Basic statistics for leave-one-out cross-validation of Plan 1

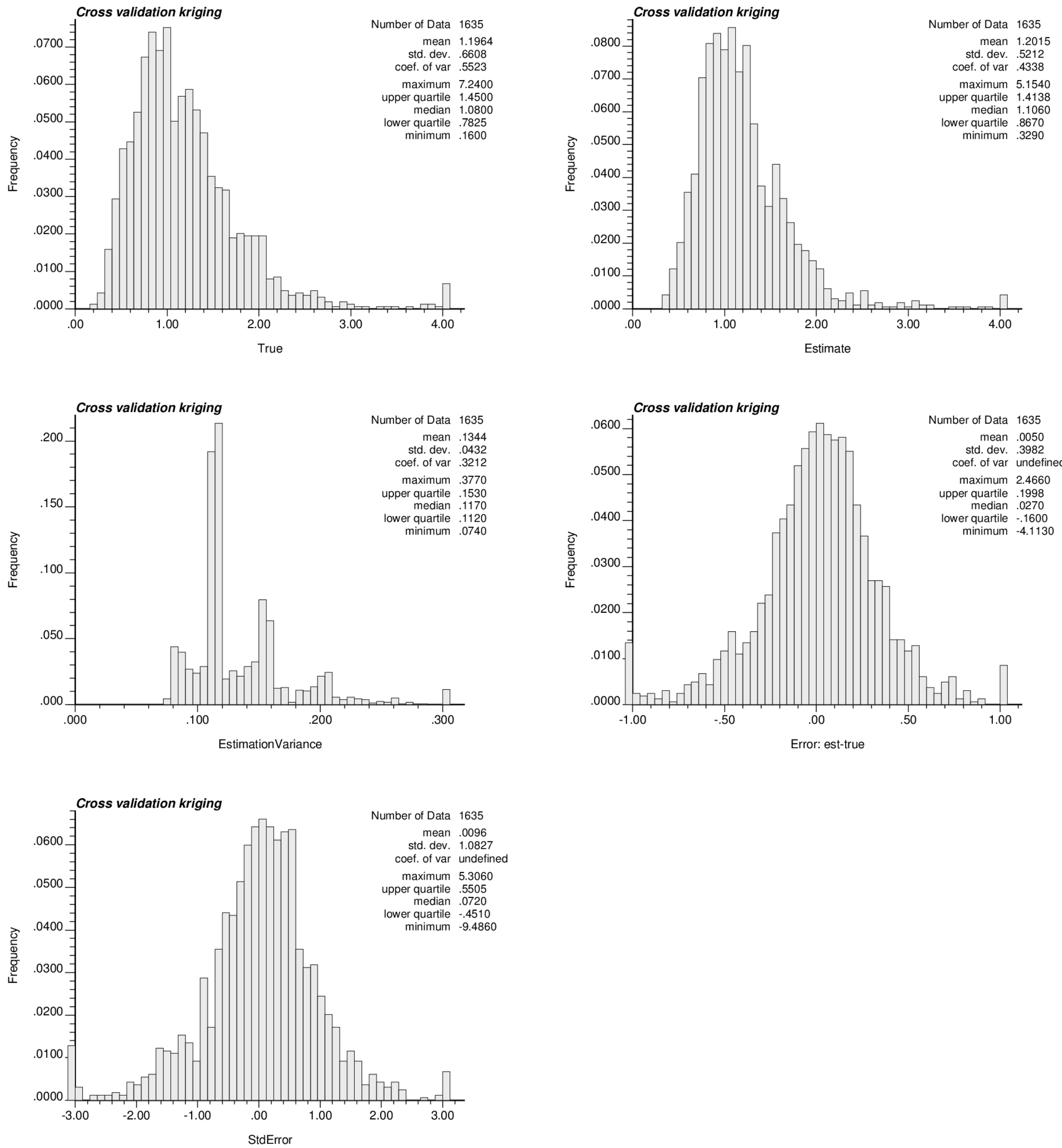


Figure 3: Leave-one-out cross validation results (histograms) for Plan 2 described in **Table 3**.

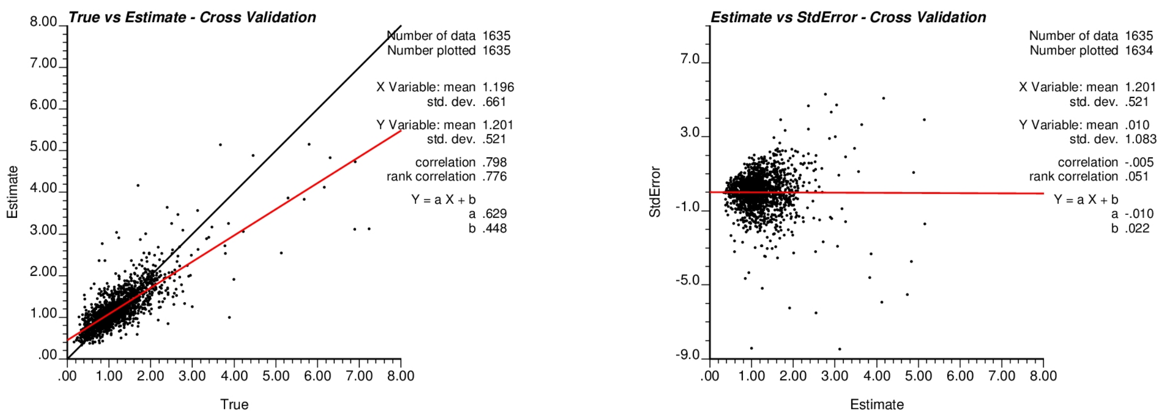


Figure 4: Leave-one-out cross validation results (scatter plots) for Plan 2 described in **Table 3**.

| | **True** | **Estimate** | **EstVar** | **Error** | **StdErr** |
|---|---|---|---|---|---|
| **Number of Data** | 1635 | 1635 | 1635 | 1635 | 1635 |
| **Mean** | 1.1964 | 1.2015 | 0.1344 | 0.0050 | 0.0096 |
| **Std. Dev.** | 0.6608 | 0.5212 | 0.0432 | 0.3982 | 1.0827 |
| **Coef. of Var.** | 0.5523 | 0.4338 | 0.3212 | undef | undef |
| **Maximum** | 7.24 | 5.15 | 0.38 | 2.47 | 5.36 |
| **Upper Quartile** | 1.45 | 1.41 | 0.15 | 0.20 | 0.55 |
| **Median** | 1.08 | 1.11 | 0.12 | 0.03 | 0.07 |
| **Lower Quartile** | 0.78 | 0.87 | 0.11 | -0.16 | -0.45 |
| **Minimum** | 0.16 | 0.33 | 0.07 | -4.11 | -9.49 |
| **Correl. Coef. True vs Estimate** | | | | 0.798 | |
| **Correl. Coef. Estimate vs StdErr** | | | | -0.005 | |

Table 5: Basic statistics for leave-one-out cross-validation of Plan 2

- Variability captured by kriging: 64%
- Variability not captured by kriging: 36%
- Variability due to nugget effect: 11%

In summary from the 89% of the variability, kriging captures 64% and 25% is not recovered.

Finally, the correlation coefficient between the estimate and the standard error is close to 0, which indicates that the standard error is independent of the estimate.

## Leave-drillhole-out cross validation

Now, we perform cross-validation leaving out the entire drillhole corresponding to the sample location estimated. The results for Plan 1 are summarized in **Figures 5** and **6**, while Plan 2 is summarized in **Figures 7** and **8**.

Again, results are as expected. **Tables 6** and **7** show the leave-drillhole-out validations for Plans 1 and 2, respectively.

As before, the estimates are unbiased (mean errors of 0.0053 and 0.0091) and in both cases the variance is reduced due to the smoothing effect of kriging: the standard deviation of the true grades is 0.6608, while that of the estimated from Plan 1 is 0.5673, and for Plan 2 0.4307. Here, we can see a **significant smoothing effect** when using 24 and 48 samples versus 2 and 4 samples. This occurs because the estimation is no longer conditioned by very close data as in the leave-one-out validation cases.

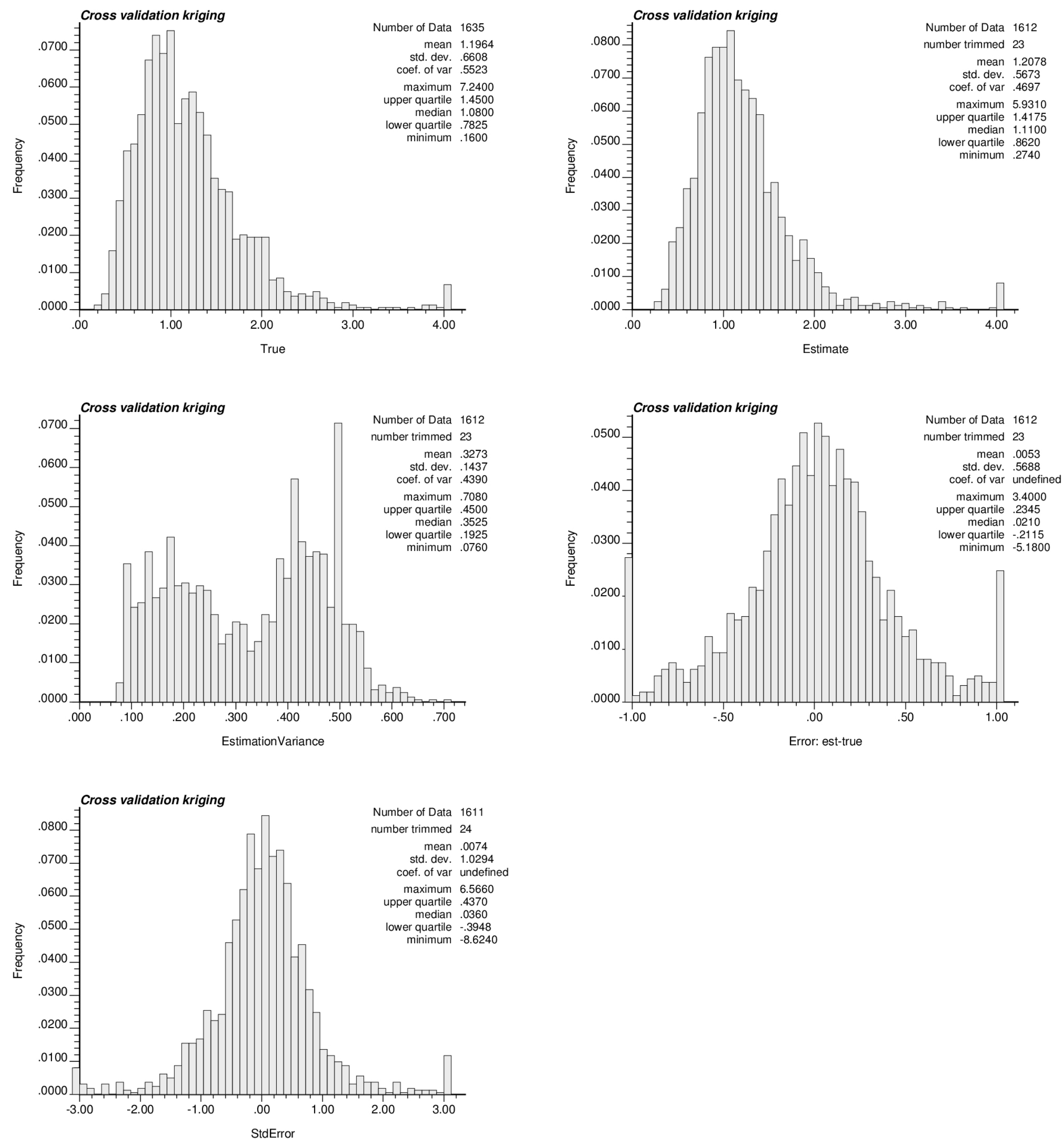


Figure 5: Leave-drillhole-out cross validation results (histograms) for Plan 1 described in **Table 3**.

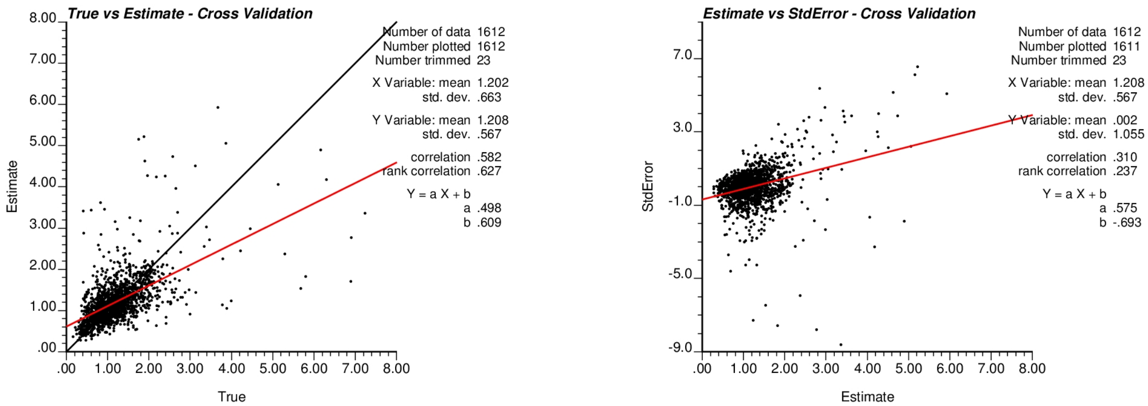


Figure 6: Leave-drillhole-out cross validation results (scatter plots) for Plan 1 described in **Table 3**.

| | **True** | **Estimate** | **EstVar** | **Error** | **StdErr** |
|---|---|---|---|---|---|
| **Number of Data** | 1635 | 1612 | 1612 | 1612 | 1611 |
| **Mean** | 1.1964 | 1.2078 | 0.3273 | 0.0053 | 0.0074 |
| **Std. Dev.** | 0.6608 | 0.5673 | 0.1437 | 0.5688 | 1.0294 |
| **Coef. of Var.** | 0.5523 | 0.4697 | 0.4390 | undef | undef |
| **Maximum** | 7.24 | 5.93 | 0.71 | 3.40 | 6.57 |
| **Upper Quartile** | 1.45 | 1.42 | 0.45 | 0.23 | 0.48 |
| **Median** | 1.08 | 1.11 | 0.35 | 0.02 | 0.03 |
| **Lower Quartile** | 0.78 | 0.86 | 0.19 | -0.21 | -0.39 |
| **Minimum** | 0.16 | 0.27 | 0.08 | -5.18 | -8.62 |
| **Correl. Coef. True vs Estimate** | | | | 0.582 | |
| **Correl. Coef. Estimate vs StdErr** | | | | 0.310 | |

Table 6: Basic statistics for leave-drillhole-out cross-validation of Plan 1

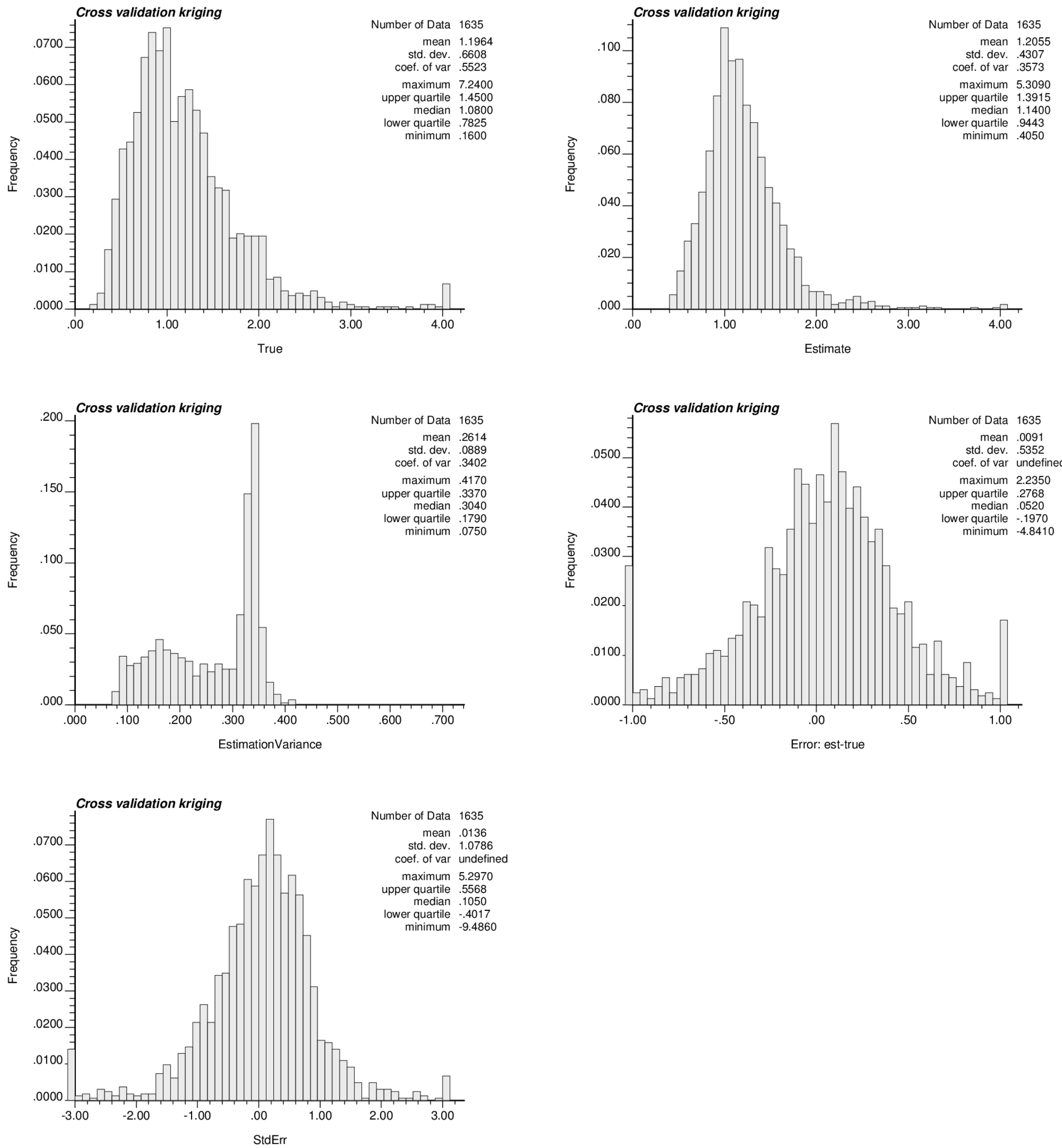


Figure 7: Leave-drillhole-out cross validation results (histograms) for Plan 2 described in **Table 3**.

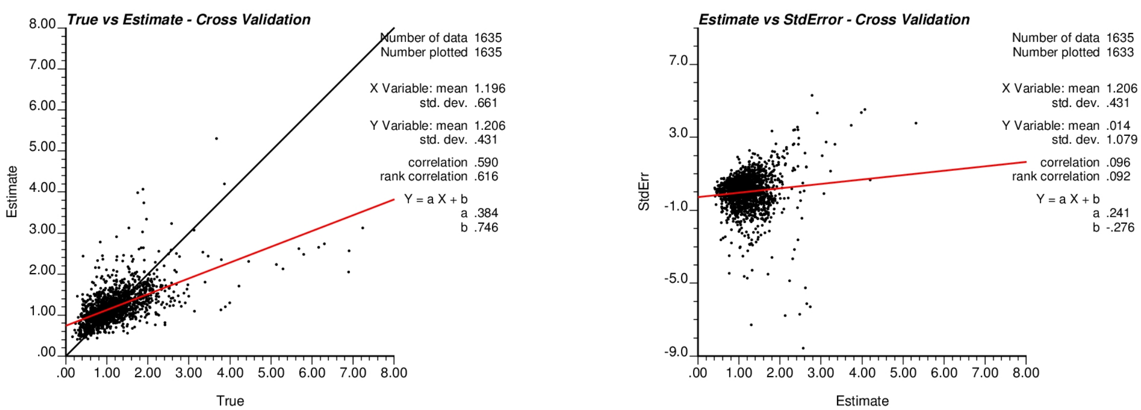


Figure 8: Leave-drillhole-out cross validation results (scatter plots) for Plan 2 described in **Table 3**.

| | **True** | **Estimate** | **EstVar** | **Error** | **StdErr** |
|---|---|---|---|---|---|
| **Number of Data** | 1635 | 1635 | 1635 | 1635 | 1635 |
| **Mean** | 1.1964 | 1.2055 | 0.2614 | 0.0091 | 0.0136 |
| **Std. Dev.** | 0.6608 | 0.4307 | 0.0889 | 0.5352 | 1.0786 |
| **Coef. of Var.** | 0.5523 | 0.3573 | 0.3402 | undef | undef |
| **Maximum** | 7.24 | 5.31 | 0.42 | 2.24 | 5.30 |
| **Upper Quartile** | 1.45 | 1.39 | 0.34 | 0.28 | 0.56 |
| **Median** | 1.08 | 1.14 | 0.30 | 0.05 | 0.11 |
| **Lower Quartile** | 0.78 | 0.94 | 0.18 | -0.20 | -0.40 |
| **Minimum** | 0.16 | 0.41 | 0.08 | -4.84 | -9.49 |
| **Correl. Coef. True vs Estimate** | | | | 0.590 | |
| **Correl. Coef. Estimate vs StdErr** | | | | 0.096 | |

Table 7: Basic statistics for leave-drillhole-out cross-validation of Plan 2

The correlation coefficients decrease between the true and estimate values, reaching 0.582 and 0.590. Here, the slight improvement in correlation in Plan 2, indicates that using more data improves the precision of the estimation (at a cost of more smoothing). This is also reflected in the lower standard deviation of the errors. The correlation coefficient between the estimate and the standard error is no longer close to 0, when using few samples for estimation.

## 5.2 Kriging

We now move on to estimate a block model. We show results not yet constrained by the domain boundaries. For illustration purposes, we start with a point estimation and then present results of block kriging.

### Point estimation

Point estimation implies that the estimated value has the same support than the sample data. Typically the sample data comes from drillholes, thus, the support will be a thin and long cylinder in shape.

We perform a dense estimation over a $1m \times 1m \times 12m$ grid. We still need to imagine that at the center of each one of those “blocks” of $1m^2$ in area and $12m$ in height, there is a volume with the sample support where the estimation has been done, hence the vertical resolution of the model is $12m$ as this is the minimum vertical support of the samples.

Several cases are computed to illustrate the effect of parameters in the estimation plan. These are presented in **Table 8**. Notice that we did not change the angles of the search ellipsoid in any case. The first three cases use the same search radii but require an increasing number of samples for estimation, starting from a minimum and maximum of 2/4 samples to 8/16 and then to a demanding plan that requires 24/48 samples (cases 1, 2, and 3). Then we fix the samples required at 8/16 but change the search radii from 25*m* to 50*m* and to 150*m*. In all cases all other parameters, including the variogram model and the use of other constraints for selecting the sample data are kept constant.

| **Parameter** | **Case 1** | **Case 2** | **Case 3** | **Case 4** | **Case 5** | **Case 6** |
|---|---|---|---|---|---|---|
| **Estimation method** | | OK | | | OK | |
| **Discretization** | | $1 \times 1 \times 1$ | | | $1 \times 1 \times 1$ | |
| **Min samples** | 2 | 8 | 24 | 8 | 8 | 8 |
| **Max samples** | 4 | 16 | 48 | 16 | 16 | 16 |
| **Max per octant** | | N/A | | | N/A | |
| **Search radii** | 50*m* | 50*m* | 50*m* | 25*m* | 50*m* | 150*m* |

Table 8: Parameters for 6 kriging plans

Representative plan views of all the cases are shown in **Figure 9**.

It can be seen that as the kriging plan is more demanding, fewer points get estimated, since many cannot find the minimum required samples for estimation within the neighborhood. It is also apparent that, as the number of samples increases, the model tends to look smoother and does not show the artifacts seen when few samples are used. Notice, in particular the circle at the bottom right in cases 1 and 2 and the discontinuities due to the abrupt change in local mean, as one sample falls in or out the search neighborhood.

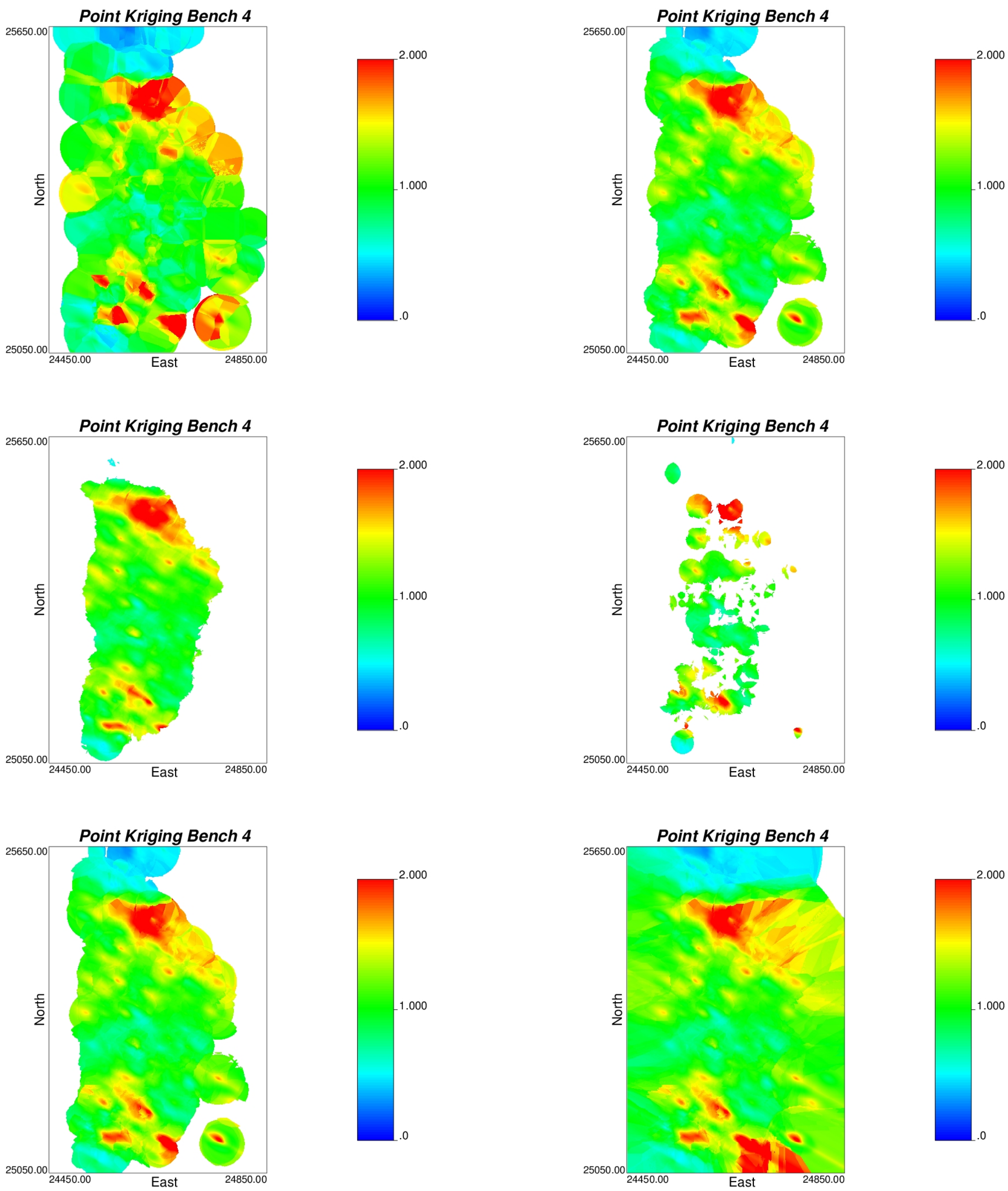


Figure 9: Plan view of bench 4 of point kriging result for the 6 cases described in **Table 8**. Top: case 1 (left), case 2 (right). Center: case 3 and case 4. Bottom: case 5 and case 6.

For cases 4 to 6, it is apparent that, as the search radius increases for a fixed minimum and maximum number of samples, more locations are estimated, as expected, without significantly changing the estimated value. Slight changes occur in locations where the estimation with a constrained search radius found a number below the maximum number of samples, 16 in this case. As the radius is increased, more samples may be added to the estimation, but their weight will be relatively low.

### Block estimation

Block kriging is run considering blocks of size $10m \times 10m \times 12m$. The discretization is $4 \times 4 \times 1$. The estimation parameters are summarized in **Table 9**.

| **Parameter** | **Block kriging** |
|---|---|
| **Estimation method** | OK |
| **Discretization** | $4 \times 4 \times 1$ |
| **Min samples** | 8 |
| **Max samples** | 16 |
| **Max per octant** | N/A |
| **Search radii** | $150m$ |

Table 9: Parameters for block kriging

Some of the plan views of the estimated model are presented in **Figure 10**.

## 5.3 Validation of the block model

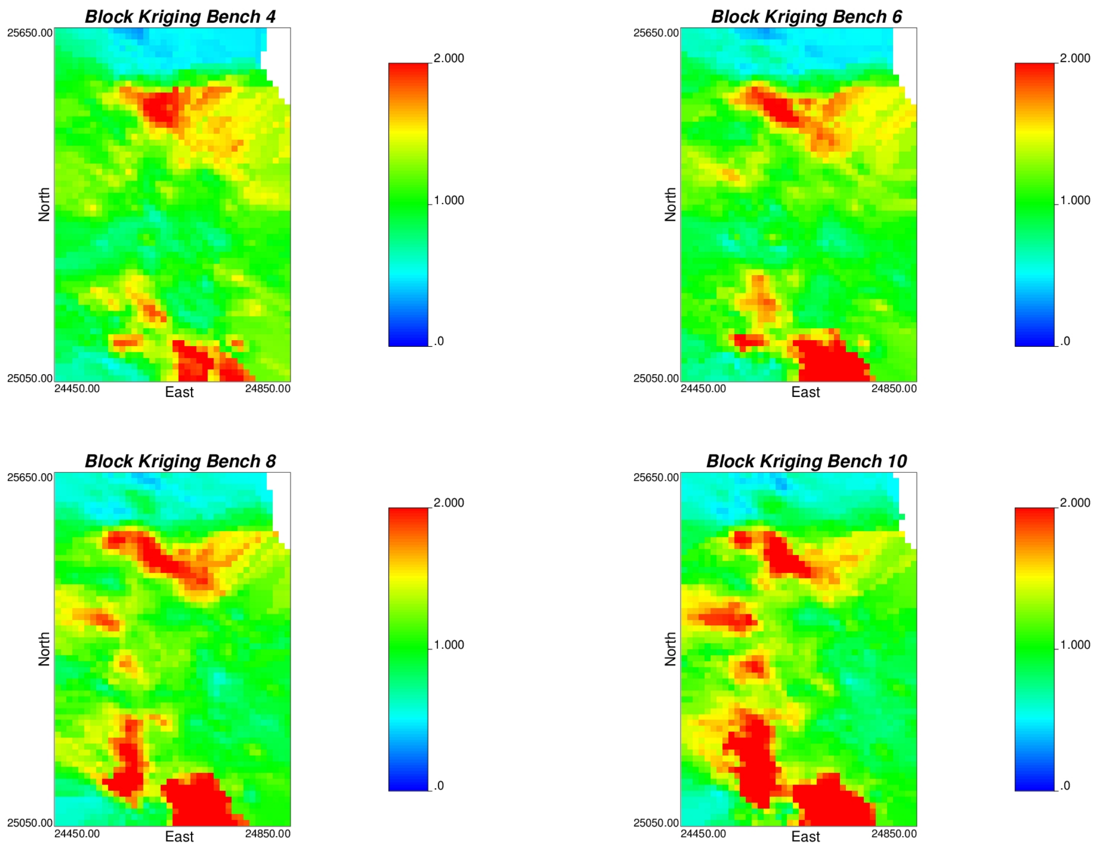


Figure 10: Block kriging plan views.

## Visual comparison

A location map of the samples within the slice corresponding to bench 6 is created and compared to the block model estimated in that bench. These images are shown in **Figure 11**. For clarity, they are superimposed in **Figure 12**.

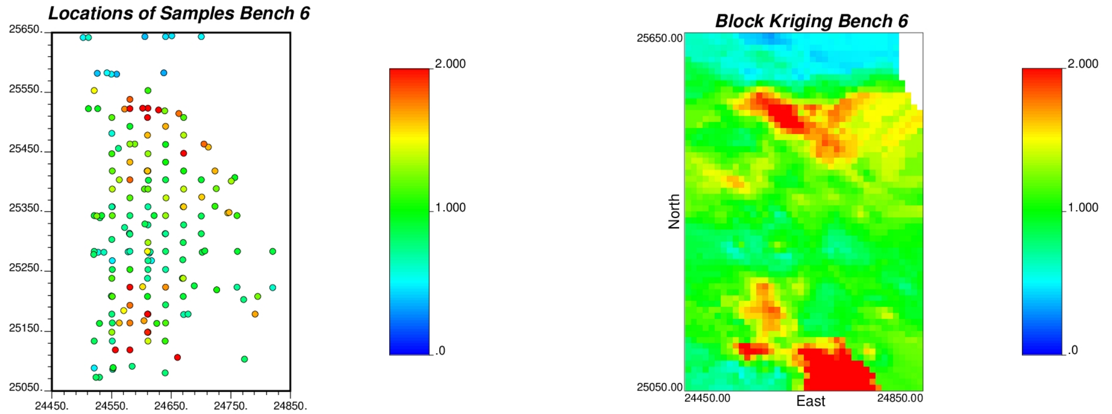


Figure 11: Location map of samples and block model obtained with kriging for bench 6.

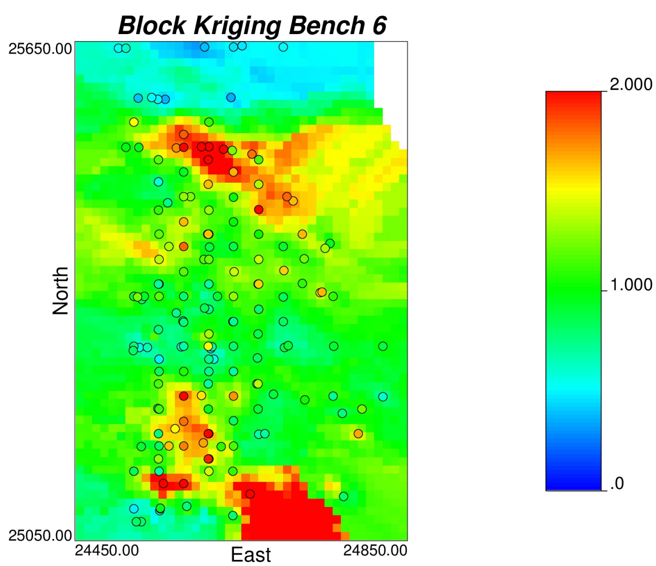


Figure 12: Location map of samples and block model obtained with kriging for bench 6, superimposed.

## Statistical comparison

For the statistical comparison, we compute the histogram of the estimated block grades (see **Figure 13**). A slight difference in means can be seen of about 0.04%Cu. Statistics are provided in **Table 10**.

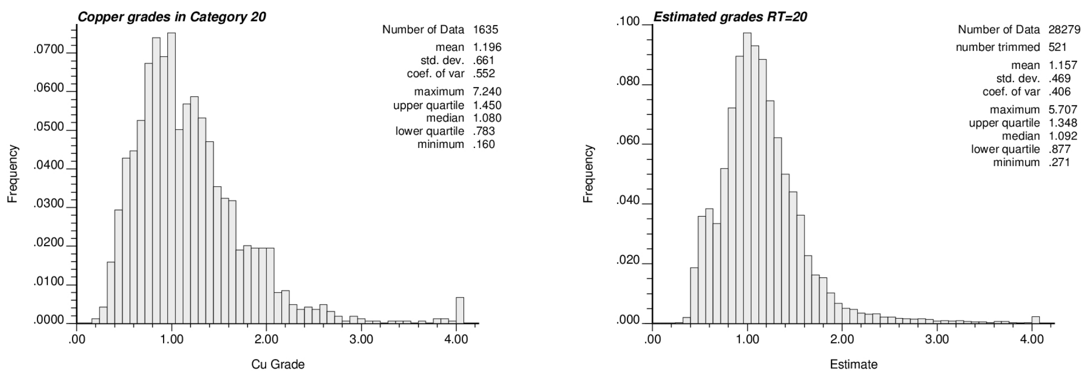


Figure 13: Histogram of samples and of estimated block grades.

## Swath plots

We complete the analysis by presenting the swath plots. These are computed using a nearest neighbor estimate (to provide declustered statistics). The conditional means in slices for the three coordinate directions (the swath plots) are presented in **Figures 14, 15** and **16**.

The main differences between the kriged model and the nearest neighbor estimate occurs in the range of East coordinates from 24700 to 24850, where few samples are available. The other swath plots show a very consistent result.

| | Samples | Kriging Estimate |
|---|---|---|
| **Number of Data** | 1635 | 28279 |
| **Not estimated** | | 521 |
| **Mean** | 1.196 | 1.157 |
| **Std. Dev.** | 0.661 | 0.469 |
| **Coef. of Var.** | 0.552 | 0.406 |
| **Maximum** | 7.24 | 5.71 |
| **Upper Quartile** | 1.45 | 1.35 |
| **Median** | 1.08 | 1.09 |
| **Lower Quartile** | 0.78 | 0.88 |
| **Minimum** | 0.16 | 0.27 |

Table 10: Statistical comparison of samples vs estimated blocks.

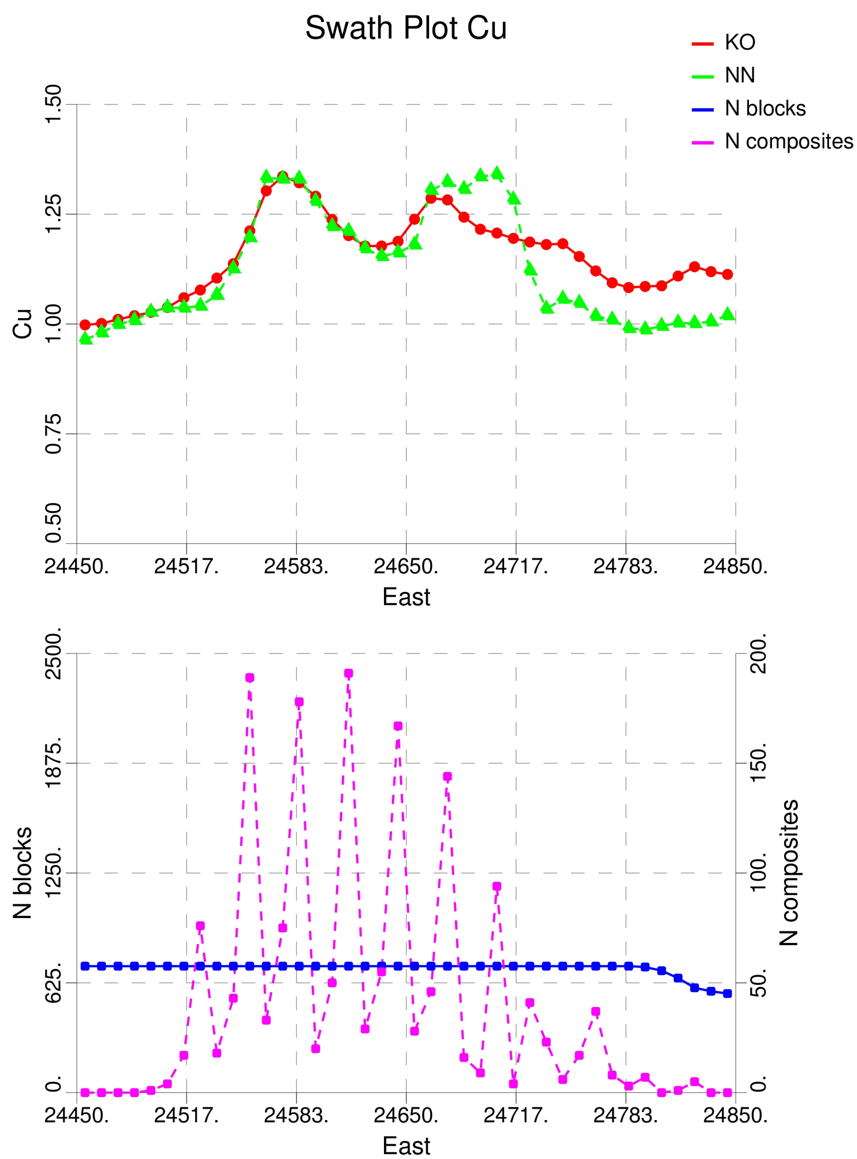


Figure 14: Swath plot in X direction.

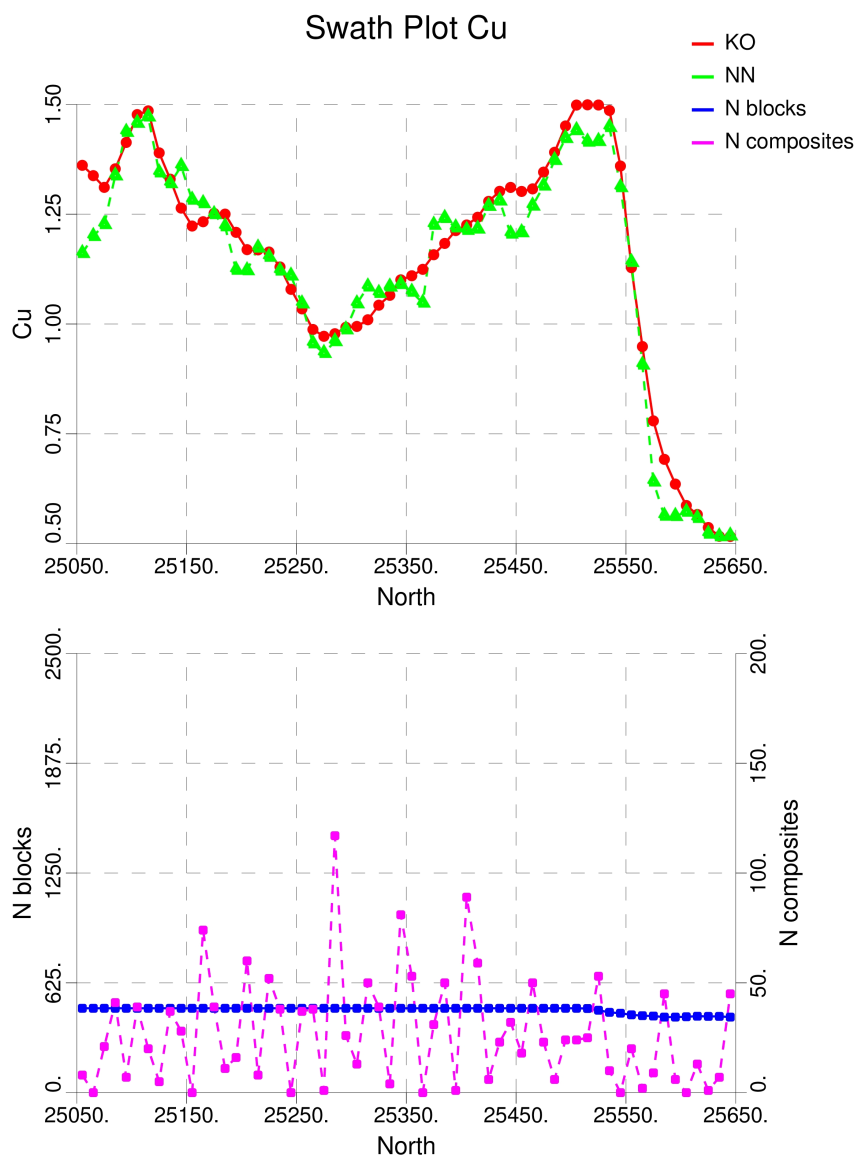


Figure 15: Swath plot in Y direction.

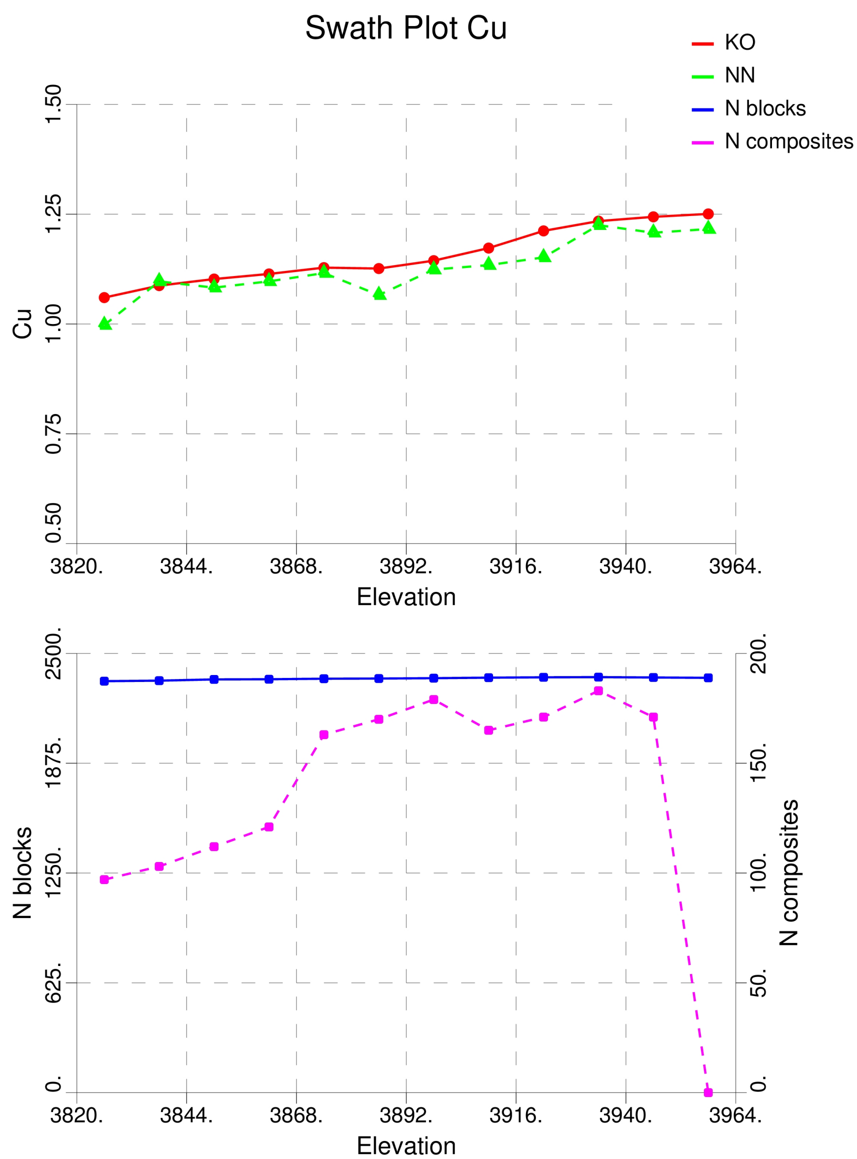


Figure 16: Swath plot in Z direction.

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